APPLICATIONS OF STEP N-FUZZY FACTOR GROUP UNDER FUZZY VERSION

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ABSTRACT

In this paper, we define the notion of Step N-Fuzzy Soft subgroup and investigate the condition under which a Fuzzy Soft subgroup is Step N-Fuzzy Soft subgroup. We introduce the notion of Step N-Fuzzy Soft cosets and establish their algebraic properties. We also initiate the study of Step N-Fuzzy Soft normal subgroups and quotient group with respect to Step N-Fuzzy Soft normal subgroup and prove some of their various group theoretic properties.

Keywords: *Soft set, Fuzzy sub group, N-Fuzzy set, Step N-Fuzzy subgroup, normal, quotient group, isomorphic, Q-Fuzzy Soft* set.

I. INTRODUCTION

The soft set theory has been applied to many different fields with great success. Maji et.al ([5], [6], [7]) worked on theoretical study of soft sets in detail, and presented an application of soft set in the decision making problem using the reduction of rough sets. The fuzzy set theory becomes a strong area of making observations in different areas like medical science, social sciences, engineering, management sciences, artificial intelligence, robotics, computer networks, decision making and so on. A fuzzy set was first introduced by Zadeh [13] and then the fuzzy sets have been used in the reconsideration of classical mathematics. Yuan et.al [12] introduced the concept of fuzzy subgroup with thresholds. Through membership function, we obtain information which makes possible for us to reach the conclusion. Due to un associated sorts of unpredictably occurring in different areas of life like economics, engineering, medical sciences, management sciences, psychology, sociology, decision making and fuzzy set as noted and often effective mathematical instruments have been offered to make, be moving in and grip those unpredictably. A fuzzy subgroup with thresholds λ and μ is also called a (λ, μ)-fuzzy subgroup. A.Solairaju and R.Nagarajan introduced the concept of structures of Q-fuzzy groups [10]. A.Solairaju and R.Nagarajan studied some structure properties of upper Q-fuzzy index order with upper Q-fuzzy subgroups [11]. A.Rosenfeld [14] defined fuzzy groups. Such inaccuracies are associated with the membership function that belongs to [0,1]. Since the establishment of fuzzy set, several extensions have been made such as Atanassov's ([1], [2], [3], [4]) work on intuitionistic fuzzy set (IFSs) was quite remarkable as he extended the concept of FSs by assigning non-membership degree say "N(x)" along with membership degree say "P(x)" with condition that $0 \leq P(x)+N(x) \leq 1$. Form last few decades, the IFS has been explored by many researchers and successfully applied to many practical fields like medical diagnosis, clustering analysis, decision making pattern recognition ([1], [2], [3], [4]). Strengthening the concept IFS suggest Pythagorean fuzzy sets which somehow enlarge the space of positive membership and negative membership by introducing some new condition that $0 \leq P2(x) + N2(x) \leq 1$. Molodtsov [8] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. In this paper, we introduce the notion of step N-intuitionistic fuzzy soft cossets and establish their algebraic properties. We define the notion of step N-intuitionistic fuzzy soft subgroup and investigate the condition under which a fuzzy soft subgroup is step N-intuitionistic fuzzy soft subgroup.

II. PRELIMINARIES AND BASIC CONCEPTS

In this section, we study some fundamental characterization of Step N-Fuzzy Soft subgroup which plays a key role in obtaining the basic group theoretic results in terms of their respective fuzzy versions. Some details of these concepts are given below which are very essential for our further discussion.

SOFT SET – 1

Definition – 1: Let X be a non-empty set 'A' mapping $A: X \times Q \rightarrow [-1, 0]$ is called a negative fuzzy soft subset (abbreviated an N-Fuzzy Soft subset) of X with Q-fuzzy version. Let 'A' and 'B' are two Step N-Fuzzy Soft subsets of a set X with Q-fuzzy version, and then the following characterizations of these fuzzy sets have been discussed in [J.N.Mordeson].

1. $A \leq B$ if and only if $A(x,e) \leq B(x,q),\ for\, all\ x \in X\ and\ q \in Q.$
2. $A = B$ if and only if $A \leq B\ and\ B \leq A.$
3. The complement of the N-Fuzzy Soft set A is AC and is defined is $A^C(x,q) = 1 - A(x,q)$
4. $(A \cap B)(x,q) = \min\{A(x,q), B(x,q)\},\ for\, all\ x \in X\ and\ q \in Q.$
5. $(A \cup B)(x,q) = \max\{A(x,q), B(x,q)\},\ for\, all\ x \in X\ and\ q \in Q.$

Definition – 2: Let A be a N-Fuzzy Soft subset with Q-fuzzy version of a set X and $\alpha \in [-1, 0]$. The set $A_\alpha = \{x \in X / A(x,q) \geq \alpha\}$ is called a level subset of N-Fuzzy Soft subset of A.

Definition – 3: Let 'A' be a N-Fuzzy Soft subset under Q-fuzzy version of a group G then 'A' is called a N-Fuzzy Soft subgroup if

(NFG–1) : $A(xy,q) \geq \min\{A(x,q), B(x,q)\},\ for\, all\ x \in X\ and\ q \in Q.$

(NFG–2) : $A(x^{-1},q) \geq A(x,q),\ for\, all\ x \in X\ and\ q \in Q.$

It is easy to show that an N-Fuzzy Soft subset under Q-fuzzy version of a group G satisfies $A(x,q) \leq A(e,q)\ and\ A(x^{-1},q) = A(x,q),\ for\, all\ x \in X\ and\ q \in Q,$ where 'e' is the identity element of G.

Proposition – 1: A function $A : G \times Q \rightarrow [-1, 0]$ is an N-Fuzzy Soft subset under Q-fuzzy version of a group G then if and only if

$A(xy^{-1},q) \geq \min\{A(x,q), A(y,q)\},\ for\, all\ x, y \in G\ and\ q \in Q.$

Proposition – 2: A function $A : G \times Q \rightarrow [-1, 0]$ is an N-Fuzzy Soft subset under Q-fuzzy version of a group G then

(i) $A(x,q) \leq A(e,q),\ for\, all\ x \in G\ and\ q \in Q,\ where\ 'e'\ is\, the\ identity\, element\, of\ G.$

(ii) $A(xy^{-1},q) = A(e,q), which\ implies\, that\ A(x,q) = A(y,q),\ for\, all\ x, y \in G\ and\ q \in Q.$

Theorem – 1: Let 'G' be a group and 'A' be a N-Fuzzy Soft subset under Q-fuzzy version of a group G then 'A' is a N-Fuzzy Soft subgroup if and only if the level subset Aα for $\alpha \in [-1,0], A(e,q) \geq \alpha$, is a subgroup of G, where 'e' is the identity of G.

Definition – 4: Let $A : G \times Q \rightarrow [-1, 0]$ be an N-Fuzzy Soft subgroup of G. 'A' is called N-Fuzzy Soft subgroup if $A(xy,q) = A(yx,q),\ for\, all\ x, y \in G\ and\ q \in Q.$

III. PROPERTIES OF STEP N-FUZZY SOFT SUBSETS

Definition – 5: Let $A : G \times Q \rightarrow [-1,0]$ be an N-Fuzzy Soft normal subgroup of G. For any $x \in G,$ the N-Fuzzy Soft set $x A : G \times Q \rightarrow [-1,0]$ defined by $(x A)(y,q) = A(x^{-1} y, q), \; for \; all \; y \in G \; and \; q \in Q$ is called left N-Fuzzy Soft coset of A. The right N-Fuzzy Soft coset of A may be defined in a similar way.

Definition – 6: Let $f : G_1 \times Q \rightarrow G_2 \times Q$ be a homomorphism from a group G1 into a group G2. Let A and B be N-Fuzzy Soft subset of G1 and G2 respectively, then $f(A) \; and \; f^{-1}(B)$ are respectively the image of N-Fuzzy Soft set A and the inverse image of N-Fuzzy Soft set B, for every y ∈ G2 defined as

$$f(A)(y,q) = \begin{cases} \sup A(x,q) / x \in f^{-1}(y,q), \; if \; f^{-1}(y,q) \neq \phi \\ 1, \qquad if \; f^{-1}(y,q) \neq \phi \end{cases}$$

For every $x \in G, \; f^{-1}(B)(x,q) = B\, f(x,q)$.

Remark – 1: It is quite evident that a group homomorphism 'f' admits the following characterizations.

(i) $f(A) f(x,q) \geq A(x,q), \; for \, every \; element \; x \in G \; and \; q \in Q.$

(ii) When 'f' is bijective map, $f(A) f(x,q) = A(x,q), \; for \, all \; x \in G \; and \; q \in Q.$

Definition – 7: A function $\delta : [-1, 0] \times [-1, 0] \rightarrow [-1, 0]$ is said to be a δ-norm if and only if 'δ' admits following properties for all a, b, c, d in $[-1, 0]$

(i) $\delta(a,b) = \delta(b,a)$

(ii) $\delta(a, \delta(b,c)) = \delta(\delta(a,b), c)$

(iii) $\delta(a,1) = \delta(1,a) = 1$

(iv) If $a \leq c \; and \; b \leq d$ then $\delta(a,b) = \delta(c,d).$

Definition – 8: Let $\delta_b : [-1, 0] \times [-1, 0] \rightarrow [-1, 0]$ be the bounded differences norm defined by $\delta_b(a,b) = \max(a + b + 1, \, 0), \; -1 \leq a \leq 0, \; -1 \leq b \leq 0$ clearly the bounded difference norm satisfies all the axioms of δ-norm.

Definition – 9: Let A be a N-Fuzzy Soft subset of a set X and $\alpha \in [-1, 0]$. The N-Fuzzy Soft set A_* of X is called Step N-Fuzzy Soft subset of X (with respect to fuzzy set A) and is defined as

$$A_*(x,q) = \delta_b(A(x,q), \alpha), \; for \; all \; x \in X, \; q \in Q.$$

Remark – 2: It is important to note that one can obtain the classical fuzzy soft subset $A(x,q)$ by choosing the value of α = – 1 in the above definition. Whereas the case become crisp for the choice α = 0. These algebraic facts lead to more that the case illustrates the Step N-Fuzzy Soft version with respect to any fuzzy soft subset for the value of α, when $\alpha \in [-1, 0]$.

Theorem – 2: Let 'A' and 'B' be any two fuzzy soft sets of X, then $(A \cap B)_* = A_* \cap B_*$

Proof: In view of definition – 9, we have

$$(A \cap B)_*(x,q) = \delta_b((A \cap B)(x,q), \alpha)$$

$$= \delta_b(\min(A(x,q), B(x,q)), \, \alpha)$$

$$= \min(\delta_b(A(x,q), B(x,q)), \alpha)$$
$$= \min(\delta_b(A(x,q),\alpha), \delta_b(A(y,q),\alpha), \alpha)$$
$$= \min(A_*(x,q), A_*(y,q))$$

This implies that, $(A \cap B)_* = A_* \cap B_*$.

Definition – 10: Let 'A' be a fuzzy soft subset of a group G and $\alpha \in [-1, 0]$, then A is called Step N-Fuzzy Soft subgroup of G. In other words, A is Step N-Fuzzy Soft subgroup under Q-Fuzzy version if A* satisfies the following:

(SNFG–1) : $A_*(xy,q) \geq \min\{A_*(x,q), A_*(y,q)\}$, *for all* $x, y \in G$ *and* $q \in Q$.

(SNFG–2) : $A_*(x^{-1},q) \geq \{A_*(x,q)\}$, *for all* $x, y \in G$ *and* $q \in Q$.

Proposition – 3: If $A : G \times Q \to [-1, 0]$ is a Step N-Fuzzy Soft subgroup of a group G, then

(i) $A_*(x,q) \leq A_*(e,q)$, *for all* $x \in G$ *and* $q \in Q$, *where* $'e'$ *is the identity element of* G.

(ii) $A_*(xy^{-1},q) = A_*(e,q)$, *which implies that* $A_*(x,q) = A_*(y,q)$, *for all* $x, y \in G$ *and* $q \in Q$.

Proof: (i) $A_*(e,q) = A_*(xx^{-1},q) \geq \min\{A_*(x,q), A_*(x^{-1},q)\}$

$= \min\{A_*(x,q), A_*(y,q)\}$

$= A_*(x,q)$

Hence, $A_*(e,q) \geq A_*(x,q)$, *for all* $x \in G$ *and* $q \in Q$.

(ii) $A_*(x,q) = A_*(xy^{-1}y,q)$

$\geq \min\{A_*(xy^{-1},q), A_*(y,q)\}$

$= \min\{A_*(e,q), A_*(y,q)\}$

$= A_*(y,q)$

Hence, $A_*(x,q) \geq A_*(y,q)$.

Similarly, $A_*(y,q) \geq A_*(x,q)$.

This implies that, $A_*(x,q) = A_*(y,q)$. *for all* $x, y \in G$.

In the following result, we establish a condition under which Step N-Fuzzy Soft subset of a group G is Step N-Fuzzy Soft subgroup.

Theorem – 3: Let A* be Step N-Fuzzy Soft subset of a group G, then A* is Step N-Fuzzy Soft subgroup under Q-Fuzzy version of G if and only if A*δ is a subgroup of G for all $\delta \leq A(e, q)$.

Proof: It is quite obvious that A* is non-empty, since A* be Step N-Fuzzy Soft subset of a group G, $A_0(x,q) \leq A_0(e,q)$, *for all* $x \in G$ *and* $q \in Q$.

But $x, y \in A_*^{\delta}$, *then* $A_*(x,q) \geq \delta$ *and* $A_*(y,q) \geq \delta$

Now, $A_*(xy^{-1},q) \geq \min(A_*(x,q), A_*(y^{-1},q))$

$= \min(A_*(x,q), A_*(y,q))$

$\geq \min\{\delta, \delta\} = \delta$

Therefore, $xy^{-1} \in A_*^{\delta}$.

Hence, A_*^{δ} is a subgroup of G.

Conversely, suppose A_*^{δ} is subgroup of G, for all $\delta \leq A_*(e,q)$

$Let\ x, y \in G\ and\ Let\ A_*(x,q) = a$

$A_*(y,q) = b,\ where\ a, b \in [0, 1]$.

Let $C = \min(a,b),\ then\ x, y \in A_*^{\delta},\ where\ C \leq A_*(e,q).$

So, by the assumption A_*^{C} is a subgroup of G.

This implies that $xy^{-1} \in A_*^{C}$ and hence $A_*(xy^{-1}, q) \geq \min(A_*(x,q),\ A_*(y,q))$

Consequently, A* is Step N-Fuzzy Soft subgroup of G.

Proposition – 4: Every fuzzy soft subgroup of a group G is Step N-Fuzzy Soft subgroup of G under Q-fuzzy version.

Proof: Let A be a fuzzy soft subgroup of a group G and let $x, y \in G.$

Then, $A_*(xy,q) = \delta_b(A(xy,q),\ \alpha)$

$\geq \delta_b(\min(A(x,q),\ A(y,q),\ \alpha))$

$= \min(\delta_b\ (A(x,q),\ A(y,q),\ \alpha))$

$= \min(\delta_b\ (A(x,q),\ \alpha),\ \delta_b\ (A(y,q),\ \alpha))$ and

$A_*(xy,q) \geq \min(A_*(x,q), A_*(y,q))$

Therefore, $A_*(x^{-1},q) = \delta_b\ (A(x^{-1},q),\ \alpha) = \delta_b\ (A(x,q),\ \alpha) = A_*(x,q)$

Consequently, A is Step N-Fuzzy Soft subgroup of G.

Remark – 3: The converse of the above Proposition need not be true.

Example – 1: Let $G = \{e, a, b, ab\},\ where\ a^2 = b^2 = e\ and\ ab = ba$ be the Klein 4-group. Let the fuzzy set A of G be defined as $A = \{\langle e, -0.2\rangle, \langle a, -0.4\rangle \langle b, -0.1\rangle \langle ab, -0.7\rangle\}$

Taking $\alpha = -0.3,$

$A_*(x,q) = \delta_b(A(x,q),\ \alpha)$

$= \max\ (A(x,q) + \alpha + 1,\ 0)$

$= \max\ (A(x,q) - 0.3 + 1,\ 0)$

$A_*(x,q) = 0,\ for\ all\ x \in G,\ q \in G.$

Moreover, we have $a^{-1} = a,\ \ b^{-1} = b\ \ and\ \ (ab)^{-1} = ab.$

This implies that 'A' is Step N-Fuzzy Soft subgroup of G. But clearly 'A' is not Fuzzy Soft subgroup of G.

Proposition – 5: Intersection of two Step N-Fuzzy Soft subgroup of a group G is also Step N-Fuzzy Soft subgroup.

Corollary – 1: The intersection of any finite number of Step N-Fuzzy Soft subgroup of a group G is also Step N-Fuzzy Soft subgroup of a group G.

Remark – 4: The union of any finite number of Step N-Fuzzy Soft subgroup of a group G need not be Step N-Fuzzy Soft subgroup of a group G.

Example – 2: Consider the group of integers Z. Define the two fuzzy soft subsets A and B of Z as follows.

$$A(x,q) = \begin{cases} -0.4, & if\ \ x = 2z \\ 0, & otherwise \end{cases}$$

$$B(x,q) = \begin{cases} -0.23, & if \ \ x=3z \\ -0.07, & otherwise \end{cases}$$

It can be easily verified that A and B are Step N-Fuzzy Soft subgroup of Z.

Now, $(A \cap B)(x,q) = \max\{A(x,q),\ B(x,q)\}$

Therefore,

$$(A \cap B)(x,q) = \begin{cases} -0.4, & if \ \ x \in 2z \\ -0.23, & if \ \ x \in 3z - 2z \\ -0.07, & otherwise \end{cases}$$

Take $x = 15 \ \ and \ \ \ y = 4, \ then$

$(A \cup B)(x,q) = -0.4 \ \ and \ \ (A \cup B)(y,q) = -0.23$

But, $(A \cup B)(x-y,q) = (A \cup B)(15-4,q)$

$= (A \cup B)(11,q) = -0.07 \ \ \ and$

$\min\left((A \cup B)(x,q),\ (A \cup B)(y,q)\right) = \min(-0.4,\ -0.23) = -0.4$

Clearly, $(A \cup B)(x-y,q) < \min\left((A \cup B)(x,q),\ (A \cup B)(y,q)\right)$

Consequently, we see that, the union of two Step N-Fuzzy Soft subgroup of a group G need not be Step N-Fuzzy Soft subgroup of a group G.

Definition – 11: Let A be Step N-Fuzzy Soft subgroup of a group G and $\alpha \in [-1,\ 0],$ then A is called Step N-Fuzzy Soft normal subgroup of G if and only if $x\mathrm{A}_* = \mathrm{A}_* x, \ \ for \ all \ \ x \in G.$

The following result leads to note that every fuzzy soft normal subgroup of a group G is Step N-Fuzzy Soft normal subgroup of G.

Proposition – 6: Every Fuzzy Soft normal subgroup of a group G is Step N-Fuzzy Soft normal subgroup of G.

Proof: Let A be a Fuzzy Soft normal subgroup of a group G then for any $x \in G,$ we have $x\mathrm{A} = \mathrm{A}x,$ which implies that $xA(g) = A\,x(g), \ \ \ for \ any \ \ g \in G.$

Then, we have $A(x^{-1}g,q) = A(gx^{-1},q)$

This implies that, $\delta_b\left(A(x^{-1}g,q),\ \alpha\right) = \delta_b\left(A(gx^{-1},q),\ \alpha\right)$

Hence, $x\mathrm{A}_* = \mathrm{A}_* x, \ \ for \ all \ \ x \in G.$

Consequently, A is Step N-Fuzzy Soft normal subgroup of a group G.

The converse of the above result need not be true.

Example – 3: Consider the dihedral group of degree 3 with finite presentation

$$G = D_3 = \langle a,\ b\ ;\ a^3 = b^2 = e,\ ba = a^2 b \rangle$$

Define the fuzzy soft subgroup of D3 by

$$A(x,q) = \begin{cases} -0.7, & if \ \ x \in \langle b \rangle \\ -0.02, & otherwise \end{cases}$$

Taking α = – 0.02, we have

$$x\,A_*(g) = \delta_b\left(A(x^{-1}y,\ \alpha)\right) = \ \delta_b\left(A(x^{-1}y, -0.2)\right) = 0 = A_* x$$

This shows that A is Step N-Fuzzy Soft normal subgroup of a group G.

$$A(a^2(ab)) = A(a^3b) = A(b) = -0.7$$
$$A((ab)a^2) = A(a(ba)a) = A(a(a^2b)a) = A(a^3ba) = A(ba) = -0.02$$

This implies that A is not fuzzy soft normal subgroup of G.

Proposition – 7: Let A be Step N-Fuzzy Soft normal subgroup with Q-fuzzy version of a group G then $A_*(y^{-1}xy, q) = A_*(x, q)$ or equivalently $A_o(xy, q) = A_*(yx, q)$ hold $for\ all\ x \in G\ and\ q \in Q.$

Proof: Since A is Step N-Fuzzy Soft normal subgroup of a group G, $xA_* = A_*x,$ holds $for\ all\ x \in G.$

This implies that $xA_*(y^{-1}, q) = A_*x(y^{-1}, q),\ for\ all\ y \in G\ and\ q \in G.$

In view of definition-11, the above relation becomes

$$\delta_b(A(x^{-1}y^{-1}, q), \alpha) = \delta_b(A(y^{-1}x^{-1}, q), \alpha),$$

This implies that, $A_*((yx)^{-1}, q) = A_*((xy)^{-1}, q)$

Consequently, $A_*(xy, q) = A_*(yx, q).$

Definition – 12: Let A be Step N-Fuzzy Soft normal subgroup of a group G, we define a set $G_{A_*} = \{x \in G / A_*(x, q) = A_*(e, q)\},$ where 'e' is the identity element of G.

The following result illustrates that the set G_{A_*} is in fact a normal subgroup of G.

Proposition – 8: Let A be Step N-Fuzzy Soft normal subgroup of a group G under Q-fuzzy version, then G_{A_*} is a normal subgroup of G.

Proof: Obviously, $G_{A_*} \neq \phi,\ for\ e \in G_{A_*}$

Let $x, y \in G_{A_*}$ be any element, then we have

$$A_*(xy^{-1}, q) \geq \min(A_*(x, q), A_*(y, q))$$
$$= \min(A_*(e, q), A_*(e, q)) = A_*(e, q).$$

This implies that, $A_*(xy^{-1}, q) \geq A_*(e, q)$

But, $A_*(xy^{-1}, q) \leq A_*(e, q)$

Therefore, $A_*(xy^{-1}, q) = A_*(e, q)$ this implies that, $xy^{-1} \in G_{A_*}$

Hence, G_{A_*} is a subgroup of G. Further, let $x \in G_{A_*}$ and $y \in G,\ we\ have$

$$A_*(y^{-1}xy^{-1}, q) = A_*(x, q) = A_*(e, q).$$

This implies that $y^{-1}xy^{-1} \in G_{A_*}$

Consequently, G_{A_*} is a normal subgroup of G.

Proposition – 9: Let 'A' be Step N-Fuzzy Soft normal subgroup of a group G, then

$$xA_* = yA_*,\ if\ and\ only\ if\ \ x^{-1}y \in G.$$
$$A_*x = A_*y,\ if\ and\ only\ if\ \ xy^{-1} \in G.$$

Proof: Suppose that $xA_* = yA_*,\ \ for\ all\ \ x, y \in G.$ In view definition – 9, the above relation yields

$$A_*\left(x^{-1}y, q\right) = \delta_b\left(A\left(x^{-1}g, q\right), \alpha\right)$$
$$= (xA_*)(y, q)$$
$$= (yA_*)(y, q)$$
$$= \delta_b\left(A\left(y^{-1}g, q\right), \alpha\right)$$
$$= \delta_b\left(A(e, q), \alpha\right)$$
$$= A_*(e, q)$$

This implies that $xy^{-1} \in G_{A_*}$

Conversely, let $xy^{-1} \in G_{A_*}$, then $A_*\left(x^{-1}y, q\right) = A_*(e, q)$

For any element $Z \in G_{A_*}$, $(xA_*)(z, q) = \delta_b\left(A\left(x^{-1}z, q\right), \alpha\right)$

$$= A_*\left(x^{-1}z, q\right)$$
$$= A_*\left(\left(x^{-1}y\right)\left(y^{-1}z\right), q\right)$$
$$\geq \min\left(A_*\left(x^{-1}y, q\right),\ A_*\left(y^{-1}z, q\right)\right)$$
$$= \min\left(A_*(e, q),\ A_*\left(y^{-1}z, q\right)\right)$$
$$= A_*\left(y^{-1}z, q\right) = (yA_*)(z, q)$$

Interchanging the roles of x and y, we get $(xA_*)(z, q) = (yA_*)(z, q),\ for\ all\ z \in G.$

Consequently, $(xA_*) = (yA_*)$.

(ii) One can prove this part analogous to (i).

Proposition – 10: Let A be Step N-Fuzzy Soft normal subgroup of a group G and x, y, u, v be an element in G. If $xA_* = uA_*,\ and\ \ yA_* = vA_*\ \ then\ xyA_* = uvA_*$.

Proof: Given $xA_* = uA_*$ and $yA_* = vA_*$, we have $x^{-1}u\ and\ \ y^{-1}v \in G_{A_*}$

Consider, $(xy)^{-1}uv = y^{-1}\left(x^{-1}u\right)\left(yy^{-1}\right)v$

$$= \left[y^{-1}\left(x^{-1}u\right)y\right]\left(y^{-1}v\right)$$

This implies that, $(xy)^{-1}uv \in G_{A_*}$

Consequently, $xyA_* = yxA_*$

IV. STEP N-FUZZY FACTOR GROUP

Definition – 13: Let 'A' be Step N-Fuzzy Soft subgroup of a group G and $\alpha \in [-1, 0].$ The right Step N-Fuzzy coset of A in G is denoted by A_*x and defined as $A_*x(g) = \delta_b\left(A\left(gx^{-1}\right), \alpha\right),\ for\ all\ x, y \in G.$

Similarly, we define the left Step N-Fuzzy cosets of A in G is denoted by xA_* and defined as $x\,A_*(g) = \delta_b\left(A\left(x^{-1}g\right), \alpha\right),\ \ for\ all\ \ x, y \in G.$

Definition – 14: Let A be Step N-Fuzzy Soft normal subgroup of a group G. The set of all Step N-Fuzzy cosets of A $\in$ G is denoted by G/A_* forms a group under the binary operation Δ defined as follows:

Let $x\,A_*,\ \ y\,A_* \in G/A_*,\ \ x\,A_*\,\Delta\ y\,A_* = \left((x\Delta y)A_*\right)$, for all $x, y \in G.$

This group is called the factor group or the quotient group of G with respect to Step N-Fuzzy Soft normal subgroup of a group A*.

Theorem – 4: The set G/A* in definition–14 form a group under the above stated binary operation Δ.

Proof: Let $A_* x_1 = A_* x_2$ and $A_* y_1 = A_* y_2,\ for\ x_1, x_2, y_1, y_2 \in G.$
Let $g \in G$ be any element of G.

$$[A_* x_1\ \Delta\ A_* y_1](g,q) = (A_* x_1\ y_1)(g,q)$$
$$= \delta_b\left(A\left(g(x_1 y_1)^{-1}, q\right), \alpha\right)$$
$$= \delta_b\left(A\left(g y_1^{-1} x_1^{-1}, q\right), \alpha\right)$$
$$= \delta_b\left(A\left(g y_1^{-1}, q\right) x_1^{-1}, \alpha\right)$$
$$= A_*\ x_1\left(g y_1^{-1}, q\right) = A_*\ x_2\left(g y_1^{-1}, q\right)$$
$$= \delta_b\left(A\left(g y_1^{-1}, q\right) x_2^{-1}, \alpha\right)$$
$$= \delta_b\left(A\left(x_2^{-1} g, q\right) y_1^{-1}, \alpha\right)$$
$$= A_*\ y_1\left(x_2^{-1} g, q\right) = A_*\ y_2\left(x_2^{-1} g, q\right)$$
$$= \delta_b\left(A\left(x_2^{-1} g, q\right) y_2^{-1}, \alpha\right)$$
$$= \delta_b\left(A\left(y_2^{-1} x_2^{-1}, q\right) g, \alpha\right)$$
$$= \delta_b\left(\left(A(x_2 y_2)^{-1}, q\right) g, \alpha\right)$$
$$= \delta_b\left(\left(A g(x_2 y_2)^{-1}, q\right), \alpha\right)$$
$$= (A_*\ x_2 y_2)(g,q)$$

This implies that Δ is well defined.

Obviously, the set G / A* admits closure and association property with respect to binary operator Δ.

Moreover, $A_*\ \Delta\ xA_* = eA_*\ \Delta\ xA_*$
$= (e \Delta x) A_* = x A_*$

Which implies that A* is the identity of G / A*. It is easy to note that the inverse of each element of G / A* exists as if for $x^{-1} A_* \in G / A_*$ such that

$$(x^{-1} A_*) \Delta (x A_*) = (x^{-1} \Delta x) A_0 = A_0$$

Consequently, (G / A_*) is a group under Δ.

Theorem – 5: Let A* be Step N-Fuzzy Soft normal subgroup of a group G, then there exists a natural epimorphism between G and G / A_* which may be defined as $x\ \alpha\ A_* x,\ x \in G$ is the Kernel of this homomorphism.

Proof: Let 'f' is a homomorphism as if for $x, y \in G$, we have

$$f(xy) = A_*\ xy = A_*\ x\, A_*\ y = f(x) f(y).$$

Obviously, 'f' is surjective as well.

Consequently, 'f' is an epimorphism from G to G / A_*.

Moreover, $Ker\, f = \{x \in G / f(x) = A_*\ e\}$
$= \{x \in G / A_*\ x = A_*\ e\}$

$= \{x \in G / x e^{-1} \in G_{A_*} \}$
$= \{x \in G / x \in G_{A_*} \}$
$= G_{A_*}$.

Theorem – 6: Let A* be Step N-Fuzzy Soft normal subgroup of a group G, then $G / A_* \cong G / G_{A*}$

Proof: In view of definition – 12, G / GA* is well defined.
Define a map, $f : G / A_* \to G / G_{A*}$ by the role $f(x A_*) = x G_{A*}$; $x \in G$
'f' is well defined because if $x A_* = y A_*$

$$x G_{A*} = x G_{A*}$$

Hence, $x A_* = y A_*$, 'f' is surjective as far each $x \in G_{A*} \in G_{A*}$, there exists $x A_* \in G / A_*$ as for each $x A_*, y A_* \in G / A_*$

$f(x A_* y A_*) = f((xy) A_*) = xy G_{A*} = x G_{A*} y G_{A*}$
$= f(x A_*) f(y A_*)$.

Consequently, there is an isomorphism between G / A_* *and* G / G_{A*}

V. HOMOMORPHISM OF STEP N-FUZZY SOFT SUBGROUP

Theorem – 7: Let $f : G_1 \to G_2$ be a bijective homomorphism from a group G1 to a group G2 and let A be Step N-Fuzzy Soft subgroup of G1 under Q-fuzzy version, then $f(A)$ is Step N-Fuzzy Soft subgroup of G2.

Proof: Let A be Step N-Fuzzy Soft subgroup of a group G. Let $y_1, y_2 \in G_2$, then there exists $x_1, x_2 \in G_1$ such that $f(x_1) = y_1$ *and* $f(x_2) = y_2$.

Consider, $(f(A))_*(y_1 y_2, q) = \delta_b(f(A)(y_1 y_2, q), \alpha)$

$$
\begin{aligned}
&= \delta_b((f(A) f(x_1) f(x_2), q), \alpha) \\
&= \delta_b((f(A) f(x_1 x_2), q), \alpha) \\
&= \delta_b((A(x_1 x_2, q)), \alpha) \\
&= A_*(x_1 x_2, q) \\
&\geq \min \{A_*(x_1, q), A_*(x_2, q)\}, \ \textit{for all } x_1 x_2 \in G_1 \\
&\geq \min \{\max \{A_*(x_1, q) / f(x_1) = y_1\}, \max \{A_*(x_2, q) / f(x_2) = y_{21}\}\} \\
&= \min (f(A_*)(y_1, q), \ f(A_*)(y_2, q)) \\
&= \min ((f(A)_*(y_1, q), \ f(A)_*(y_2, q)))
\end{aligned}
$$

Moreover, $(f(A))_*(y^{-1}, q) = f(A_*)(y^{-1}, q)$

$$
\begin{aligned}
&= \max \{A_*(x^{-1}, q) / \ f(x^{-1}) = y^{-1}\}, \\
&= \max \{A_*(x, q) / \ f(x) = y\}, \\
&= (f(A))_*(y, q).
\end{aligned}
$$

Consequently, $f(A)$ is Step N-Fuzzy Soft subgroup of a group G.

Theorem – 8: Let $f : G_1 \rightarrow G_2$ be a bijective homomorphism from a group G1 to a group G2 and let A be Step N-Fuzzy Soft subgroup of G1, then $f(A)$ is Step N-Fuzzy Soft normal subgroup of G2.

Proof: In view of Theorem – 7, it is sufficient to show that $f(A_*)$ is fuzzy soft normal in G2.

Let A be Step N-Fuzzy Soft normal subgroup of G1, Let $y_1, y_2 \in G_2$, then there exists $x_1, x_2 \in G_1$ such that $f(x_1) = y_1 \; and \; f(x_2) = y_2$.

Consider,
$$
\begin{aligned}
(f(A))_*(y_1 y_2, q) &= \delta_b(f(A)(y_1 y_2, q), \alpha) \\
&= \delta_b((f(A) f(x_1) f(x_2), q), \alpha) \\
&= \delta_b((f(A) f(x_1 x_2), q), \alpha) \\
&= \delta_b((A(x_1 x_2, q)), \alpha) \\
&= A_*(x_1 x_{21}, q) \\
&= \delta_b((A(x_2 x_1, q)), \alpha) \\
&= \delta_b((f(A) f(x_2 x_1), q), \alpha) \\
&= \delta_b((f(A) f(x_2) f(x_1), q), \alpha) \\
&= \delta_b(f(A)(y_2 y_1, q), \alpha) \\
&= (f(A))_*(y_2 y_1, q)
\end{aligned}
$$

Consequently, $f(A)$ is Step N-Fuzzy Soft normal subgroup of G2.

Theorem – 9: Let $f : G_1 \rightarrow G_2$ be a bijective homomorphism from a group G1 to a group G2 and let 'B' be Step N-Fuzzy Soft subgroup of G2, then $f^{-1}(B)$ is Step N-Fuzzy Soft subgroup of G1.

Proof: Let 'B' be Step N-Fuzzy Soft subgroup of G2, Let $x_1, x_2 \in G_1$,
$$
\begin{aligned}
(f^{-1}(B))_*(x_1 x_2, q) &= B_*(f(x_1 x_2), q) \\
&= B_*(f(x_1) f(x_2), q) \\
&\geq \min(B_*(f(x_1), q), B_*(f(x_2), q)) \\
&= \min(f^{-1}(B_*)(x_1, q), f^{-1}(B_*)(x_2, q)) \\
&= \min((f^{-1}(B))_*(x_1, q), (f^{-1}(B))_*(x_2, q))
\end{aligned}
$$

Further,
$$
\begin{aligned}
(f^{-1}(B))_*(x^{-1}, q) = f^{-1}(B_*)(x^{-1}, q) &= B_*(f(x^{-1}), q) \\
&= B_*((f(x))^{-1}, q) \\
&= B_*((f(x)), q) \\
&= (f^{-1}(B))_*(x, q)
\end{aligned}
$$

Consequently, $f^{-1}(B)$ is Step N-Fuzzy Soft subgroup of G1.

Theorem – 10: Let $f : G_1 \rightarrow G_2$ be a bijective homomorphism from a group G1 to a group G2 and let B be Step N-Fuzzy Soft normal subgroup of G2, then $f^{-1}(B)$ is Step N-Fuzzy Soft normal subgroup of G1.

Proof: In view of Theorem – 9, it is sufficient to show that $f^{-1}(B)$ is fuzzy soft normal in G1.

Let B be Step N-Fuzzy Soft normal subgroup of G2 Let $x_1, x_2 \in G_1$, then we have

$$\left(f^{-1}(B)\right)_*(x_1x_2\,,\,q) = B_*\left(f(x_1x_2),\,q\right)$$
$$= B_*\left(f(x_1)\,f(x_2),\,q\right)$$
$$= B_*\left(f(x_2)\,f(x_1),\,q\right)$$
$$= \left(f^{-1}(B)\right)_*(x_2x_1\,,\,q)$$

Consequently, $f^{-1}(B)$ is Step N-Fuzzy Soft normal subgroup of G1.

VI. CONCLUSION

In this paper, we have introduced the concept of Step N-Fuzzy Soft subgroup and Step N-Fuzzy Soft coset a given group and has used them to introduce the concept of Step N-Fuzzy Soft normal subgroup and have discussed various related properties. We have also studied the effect on the image and inverse image of Step N-Fuzzy Soft subgroup (normal) under group homomorphism.

VII. FUTURE WORK

We shall extend this idea to intuitionistic fuzzy sets, vague soft sets and will investigate its various algebraic structures.

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