Exploring a Generalized Hypergeometric Function Approach in a Modified Gamma Distribution Model

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ABSTRACT

In the present paper, the author has studied about the structures which are the products and ratios of statistically independently distributed positive real scalar random variables.The author has derived the exact density of Generalized Gamma density by the Mellin Transform and Hankel Tranform of the unknown density and afterthat the unknown density has been derived in terms of H - functions by taking the inverse Mellin transform and Inverse Hankel Transform .A more general structure of generallized Gamma density has also been discussed.

Keywords: Generalized Gamma Density, Wright's Generalized Hypergeometric Function, H -function, Mellin Transform, Inverse Mellin Transform, Hankel Transform, Inverse Hankel Transform.
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I. INTRODUCTION

Generalized Wright's function ${}_2R_1(a,b;c,w;\mu;z)$ defined by Dotsenko [1, 2] hs been denoted as

$$ {}_2R_1(a,b;c,w;\mu;z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{k=0}^{\infty}\frac{\Gamma(a+k)\Gamma\left(b+k\frac{w}{\mu}\right)}{\Gamma\left(c+k\frac{w}{\mu}\right)}\frac{z^k}{k!} $$

$$ =\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\,{}_2\psi_1\left[z\Big|_{\left(c,\frac{w}{\mu}\right)}^{(a,1),\left(b,\frac{w}{\mu}\right)}\right] \qquad (1.1) $$

The H -function is defined by means of a Mellin-Barnes type integral in the following manner (Mathai and Saxena, 1978):

$$ H(z)=H_{p,q}^{m,n}(z)=H_{p,q}^{m,n}\left[z\Big|_{(b_q,B_q)}^{(a_p,A_p)}\right] $$

$$ =H_{p,q}^{m,n}\left[z\Big|_{(b_1,B_1),\dots,(b_q,B_q)}^{(a_1,A_1),\dots,(a_p,A_p)}\right]=\frac{1}{2\pi i}\int_L \theta(s)z^{-s}ds \qquad (1.2) $$

Where $i=\sqrt{-1},\ z\neq 0$ and $z^{-s}=\exp[-\sin|z|+i\arg z]$ where $|z|$ represents the natural logarithim of $|z|$ and $\arg z$ is not the principal value. Here

$$ \theta(s)=\frac{\prod_{j=1}^{m}\Gamma\left(b_j+B_js\right)\prod_{j=1}^{n}\Gamma\left(1-a_j-A_js\right)}{\prod_{j=m+1}^{q}\Gamma\left(1-b_j-B_js\right)\prod_{j=n+1}^{p}\Gamma\left(a_j+A_js\right)} \qquad (1.3) $$

The Mellin transform of $f(x)$ denoted by $M\{f(x);s\}$ or $F(s)$ is given by

$$ M\{f(x);s\}=\int_0^{\infty}x^{s-1}f(x)dx \qquad (1.4) $$

The Hankel transform of $f(x)$ denoted by $H_v\{f(x);p\}$ or $F_v(p)$ is given by

$$H_v\left\{f(x);p\right\}=\int_0^\infty x\,J_v(px)\,f(x)dx \tag{1.5}$$

II. GENERAL STRUCTURES

A real scalar random variable x is said to have a generalized gamma distribution, when the density is of the following form:

$$f(x)=\begin{cases}\dfrac{\beta A^{\frac{\alpha+tm}{\beta}}}{\Gamma\left(\frac{\alpha+tm}{\beta}\right){}_2R_1(a,b;c,w;\mu;p)}x^{\alpha-1}e^{-ax^{\beta}}\,{}_2R_1(a,b;c,w;\mu;px^t);x>0,A>0,\alpha>0,\beta>0\\ 0,elsewhere\end{cases} \tag{2.1}$$

Where the parameters α and β are real. The following discussion holds even when α and β are complex quantities. In this case, the conditions become $\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0$ where Re(.)means the real part of (.).

Let E denote the mathematical expectation, the h^{th} -moment of x, when x has the density in (2.1), is given by

$$E\left(x^h\right)=\frac{1}{A^{\frac{h}{\beta}}}\frac{\Gamma\left(\frac{\alpha+tm+h}{\beta}\right)}{\Gamma\left(\frac{\alpha+tm}{\beta}\right)} \tag{2.2}$$

For $\mathrm{Re}(\alpha+tm+h)>0$.

When α and h are real, the moments can exist for some values of h also such that $\alpha+h>0$.

The Mellin transform of $f(x)$ is obtained from (2.2) as:

$$M\left\{f(x)\right\}=E\left(x^{s-1}\right)=\frac{1}{A^{\frac{s-1}{\beta}}}\frac{\Gamma\left(\frac{\alpha+tm+s-1}{\beta}\right)}{\Gamma\left(\frac{\alpha+tm}{\beta}\right)} \tag{2.3}$$

For $\mathrm{Re}(\alpha+tm+s-1)>0, s=v+2r+2$.

The unknown density $f(x)$ is obtained in terms of H -function by taking the inverse Mellin transform of (2.3). That is

$$f(x)=\frac{1}{A^{\frac{-1}{\beta}}\Gamma\left(\frac{\alpha+tm}{\beta}\right)}H_{0,1}^{1,0}\left[A^{\frac{1}{\beta}}x\left|\begin{matrix}---\\ \left(\frac{\alpha+tm-1}{\beta},\frac{1}{\beta}\right)\end{matrix}\right.\right] \tag{2.4}$$

The Hankel transform of $f(x)$ is obtained from (2.2) as:

$$H\left\{f(x)\right\}=E\left(xJ_{v}(px)\right)=J_{v}(p)\frac{1}{A^{\frac{s-1}{\beta}}}\frac{\Gamma\left(\frac{\alpha+tm+s-1}{\beta}\right)}{\Gamma\left(\frac{\alpha+tm}{\beta}\right)} \tag{2.5}$$

For $\operatorname{Re}(\alpha+tm+s-1)>0, s=v+2r+2$.

The unknown density $f(x)$ is obtained in terms of H-function by taking the inverse Hankel transform of (2.5). That is

$$f(x)=J_{v}(p)\frac{1}{A^{\frac{-1}{\beta}}\Gamma\left(\frac{\alpha+tm}{\beta}\right)}H_{0,1}^{1,0}\left[A^{\frac{1}{\beta}}x\middle|_{\left(\frac{\alpha+tm-1}{\beta},\frac{1}{\beta}\right)}^{---}\right] \tag{2.6}$$

Consider a set of real scalar random variables $x_1,...,x_k$, mutually independently distributed, where x_j has the density in (2.1) with the parameters $\alpha_j,\beta_j; j=1,...,k$ and consider the product

$$u=x_1x_2...x_k \tag{2.7}$$

In the standard terminology in statistical literature, the h^{th} moment of u, when u has the density in (2.1), is given by

$$E\left(u^{h}\right)=\prod_{j=1}^{k}\frac{1}{A^{\frac{h}{\beta_j}}}\frac{\Gamma\left(\frac{\alpha_j+tm+h}{\beta_j}\right)}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)} \tag{2.8}$$

For $\operatorname{Re}(\alpha_j+tm+h)>0; j=1,...,k$

Then the Mellin transform of $g(u)$ of u is obtained from the property of the statistical independent and is given by

$$M\left[g(u)\right]=M\left[x_1^{s-1}\right]...M\left[x_k^{s-1}\right] \tag{2.9}$$

$$E\left(u^{s-1}\right)=\prod_{j=1}^{k}\frac{1}{A^{\frac{s-1}{\beta_j}}}\frac{\Gamma\left(\frac{\alpha_j+tm+s-1}{\beta_j}\right)}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)} \tag{2.10}$$

For $\operatorname{Re}(\alpha_j+tm+s-1)>0, s=v+2r+2$

The unknown density $f(x)$ is obtained in terms of H-function by taking the inverse Mellin transform of (2.10). That is

$$g(u)=\frac{1}{A^{\frac{-1}{\beta}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}H_{0,k}^{k,0}\left[\prod_{j=1}^{k}A^{\frac{1}{\beta}}u\left|_{\left(\frac{\alpha_j+tm-1}{\beta_j},\frac{1}{\beta_j}\right)}^{---};j=1,...,k\right.\right] \quad (2.11)$$

For $\beta_j=1; j=1,...,k$, the H -function reduces to the G -function.

Then the Hankel transform of $g(u)$ of u is obtained from the property of the statistical independent and is given by:

$$H[uJ_v(pu)]=H\left[x_1J_v(px_1)\right]H\left[x_2J_v(px_2)\right]...H\left[x_kJ_v(px_k)\right] \quad (2.12)$$

$$=J_v(p)\prod_{j=1}^{k}\frac{1}{A^{\frac{s-1}{\beta_j}}}\frac{\Gamma\left(\frac{\alpha_j+tm+s-1}{\beta_j}\right)}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)} \quad (2.13)$$

For $\mathrm{Re}(\alpha_j+tm+s-1)>0, s=v+2r+2$

The unknown density $f(x)$ is obtained in terms of H -function by taking the inverse Hankel transform of (2.13). That is

$$g(u)=J_v(p)\prod_{j=1}^{k}\frac{1}{A^{\frac{-1}{\beta_j}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}H_{0,k}^{k,0}\left[\prod_{j=1}^{k}A^{\frac{1}{\beta_j}}u\left|_{\left(\frac{\alpha_j+tm-1}{\beta_j},\frac{1}{\beta_j}\right)}^{---};j=1,...,k\right.\right] \quad (2.14)$$

For $\beta_j=1; j=1,...,k$, the H -function reduces to the G -function.

If we consider more general structures in the same category. For example, consider the structure

$$u_1=x_1^{\gamma_1}x_2^{\gamma_2}...x_k^{\gamma_k},\gamma_k>0, j=1,...,k \quad (2.15)$$

Where $x_1,...,x_k$ are mutually independently distributed as in (2.5).

Then the Mellin transform of $g(u_1)$ of u_1 is given as

$$M\left[g(u_1)\right]=M\left[x_1^{\gamma_1(s-1)}\right]...M\left[x_k^{\gamma_k(s-1)}\right] \quad (2.16)$$

$$E\left(u_1^{s-1}\right)=\prod_{j=1}^{k}\frac{1}{A^{\frac{\gamma_j(s-1)}{\beta_j}}}\frac{\Gamma\left(\frac{\alpha_j+tm+\gamma_j(s-1)}{\beta_j}\right)}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)} \quad (2.17)$$

For $\mathrm{Re}(\alpha_j+tm+s-1)>0, s=v+2r+2,\gamma_j>0$.

The unknown density $g(u_1)$ is obtained in terms of H -function by taking the inverse Mellin transform of (2.17). That is

$$g(u_1)=\frac{1}{A^{\frac{-\gamma_j}{\beta}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}H_{0,k}^{k,0}\left[\prod_{j=1}^{k}A^{\frac{\gamma_j}{\beta}}u_1\middle|_{\left(\frac{\alpha_j+tm-\gamma_j}{\beta_j},\frac{\gamma_j}{\beta_j}\right)}^{---};j=1,\ldots,k\right] \qquad (2.18)$$

For $\beta_j=1=\gamma_j;j=1,\ldots,k$, the H -function reduces to the G -function.

Then the Hankel transform of $g(u_1)$ of u_1 is obtained from the property of the statistical independent and is given by:

$$H[u_1J_v(pu_1)]=H\left[x_1^{\gamma_1}J_v(px_1^{\gamma_1})\right]H\left[x_2^{\gamma_1}J_v(px_2^{\gamma_2})\right]\ldots H\left[x_k^{\gamma_1}J_v(px_k^{\gamma_k})\right] \qquad (2.19)$$

$$==J_v(p)\prod_{j=1}^{k}\frac{1}{A^{\frac{\gamma_j(s-1)}{\beta_j}}}\frac{\Gamma\left(\frac{\alpha_j+tm+\gamma_j(s-1)}{\beta_j}\right)}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)} \qquad (2.20)$$

For $\text{Re}(\alpha_j+tm+s-1)>0,s=v+2r+2,\gamma_j>0$

The unknown density $g(u_1)$ is obtained in terms of H -function by taking the inverse Hankel transform of (2.20). That is

$$g(u_1)=J_v(p)\prod_{j=1}^{k}\frac{1}{A^{\frac{-\gamma_j}{\beta_j}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}H_{0,k}^{k,0}\left[\prod_{j=1}^{k}A^{\frac{\gamma_j}{\beta_j}}u_1\middle|_{\left(\frac{\alpha_j+tm-\gamma_j}{\beta_j},\frac{\gamma_j}{\beta_j}\right)}^{---};j=1,\ldots,k\right] \qquad (2.21)$$

For $\text{Re}(\alpha_j+tm+s-1)>0,s=v+2r+2,\gamma_j>0$

For $\beta_j=1=\gamma_j;j=1,\ldots,k$, the H -function reduces to the G -function.

A More General Structure
We can consider more general structures. Let

$$w=\frac{x_1,x_2,\ldots,x_r}{x_{r+1},\ldots,x_k} \qquad (2.22)$$

Where $x_1,\ldots,x_k$, mutually independently distributed real random variables having the density in (2.1) with x_j having parameters $\alpha_j,\beta_j;j=1,\ldots,k$.

Then the Mellin transform of $g(w)$ is given by

$$M\left[g(w)\right]=M\left[x_1^{s-1}\right]...M\left[x_r^{s-1}\right]M\left[x_{r+1}^{-(s-1)}\right]...M\left[x_k^{-(s-1)}\right] \quad (2.23)$$

$$=\prod_{j=1}^{k}\frac{A^{\frac{1}{\beta_j}}}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}\left\{\prod_{j=1}^{r}\frac{\Gamma\left(\frac{\alpha_j+tm+s-1}{\beta_j}\right)}{A^{\frac{s}{\beta_j}}}\right\}\left\{\prod_{j=r+1}^{k}\frac{\Gamma\left(\frac{\alpha_j+tm-s+1}{\beta_j}\right)}{A^{\frac{-s}{\beta_j}}}\right\} \quad (2.24)$$

For $\mathrm{Re}(\alpha_j+tm\pm(s-1))>0, s=v+2r+2.$

The unknown density $g(w)$ is obtained in terms of H-function by taking the inverse Mellin transform of (2.24). That is

$$g(w)=\prod_{j=1}^{k}\frac{1}{A^{\frac{-1}{\beta}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}\left\{H_{0,r}^{r,0}\left[\prod_{j=1}^{r}A^{\frac{1}{\beta}}w\left|\begin{matrix}---\\ \left(\frac{\alpha_j+tm-1}{\beta_j},\frac{1}{\beta_j}\right)\end{matrix}\right.;j=1,...,r\right]\right\}$$

$$\left\{H_{k-r,0}^{0,k-r}\left[\prod_{j=r+1}^{k}A^{-\frac{1}{\beta}}w\left|\begin{matrix}\left(1-\frac{\alpha_j+tm+1}{\beta_j},\frac{1}{\beta_j}\right)\\ ---\end{matrix}\right.;j=r+1,...,k\right]\right\} \quad (2.25)$$

For $\mathrm{Re}(\alpha_j+tm\pm(s-1))>0, s=v+2r+2.$

For $\beta_j=1=\gamma_j; j=1,...,k$, the H-function reduces to the G-function.

The Hankel transform of $g(w)$ is given as:

$$H[wJ_v(pw)]=H\left[x_1J_v(px_1)\right]...H\left[x_rJ_v(px_r)\right]$$
$$H\left[x_{r+1}^{-1}J_v\left(px_{r+1}^{-1}\right)\right]...H\left[x_k^{-1}J_v\left(px_k^{-1}\right)\right] \quad (2.26)$$

$$=J_v(p)\prod_{j=1}^{k}\frac{A^{\frac{1}{\beta_j}}}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}\left\{\prod_{j=1}^{r}\frac{\Gamma\left(\frac{\alpha_j+tm+s-1}{\beta_j}\right)}{A^{\frac{s}{\beta_j}}}\right\}\left\{\prod_{j=r+1}^{k}\frac{\Gamma\left(\frac{\alpha_j+tm-s+1}{\beta_j}\right)}{A^{\frac{-s}{\beta_j}}}\right\} \quad (2.27)$$

For $\mathrm{Re}(\alpha_j+tm\pm(s-1))>0, s=v+2r+2.$

The unknown density $g(w)$ is obtained in terms of H-function by taking the inverse Hankel transform of (2.27). That is

$$g(w)=J_v(p)\prod_{j=1}^{k}\frac{1}{A^{\frac{-1}{\beta}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}\left\{H_{0,r}^{r,0}\left[\prod_{j=1}^{r}A^{\frac{1}{\beta}}w\left|_{\left(\frac{\alpha_j+tm-1}{\beta_j},\frac{1}{\beta_j}\right)}^{---};j=1,...,r\right.\right]\right\}$$

$$\left\{H_{k-r,0}^{0,k-r}\left[\prod_{j=r+1}^{k}A^{-\frac{1}{\beta}}w\left|_{---}^{\left(1-\frac{\alpha_j+tm+1}{\beta_j},\frac{1}{\beta_j}\right)};j=r+1,...,k\right.\right]\right\} \quad (2.28)$$

For $\mathrm{Re}(\alpha_j+tm\pm(s-1))>0, s=v+2r+2.$

For $\beta_j=1=\gamma_j; j=1,...,k$, the H-function reduces to the G-function.

Now, we consider more general structures in the same category. For example, consider the structure

$$w_1=\frac{x_1^{\gamma_1},...,x_r^{\gamma_r}}{x_{r+1}^{\gamma_{r+1}},...,x_k^{k}} \quad (2.29)$$

Where $x_1,...,x_k$, mutually independently distributed real random variables having the density in (2.1) with x_j having parameters $\alpha_j,\beta_j; j=1,...,k$.

Then the Mellin transform of $g(w_1)$ is given by

$$M[g(w_1)]=M\left[x_1^{\gamma_1(s-1)}\right]...M\left[x_r^{\gamma_r(s-1)}\right]M\left[x_{r+1}^{-\gamma_{r+1}(s-1)}\right]...M\left[x_k^{-\gamma_k(s-1)}\right] \quad (2.30)$$

$$=\prod_{j=1}^{k}\frac{A^{\frac{\delta_j}{\beta_j}}}{\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}\left\{\prod_{j=1}^{r}\frac{\Gamma\left(\frac{\alpha_j+tm+\delta_j(s-1)}{\beta_j}\right)}{A^{\frac{s\delta_j}{\beta_j}}}\right\}\left\{\prod_{j=r+1}^{k}\frac{\Gamma\left(\frac{\alpha_j+tm-\delta_j(s-1)}{\beta_j}\right)}{A^{\frac{-s\delta_j}{\beta_j}}}\right\} \quad (2.31)$$

For $\mathrm{Re}(\alpha_j+tm\pm(s-1))>0, s=v+2r+2, \delta_j>0$.

The unknown density $g(w_1)$ is obtained in terms of H-function by taking the inverse Mellin transform of (2.31). That is

$$g(w_1)=\prod_{j=1}^{k}\frac{1}{A^{\frac{-\delta_j}{\beta}}\Gamma\left(\frac{\alpha_j+tm}{\beta_j}\right)}\left\{H_{0,r}^{r,0}\left[\prod_{j=1}^{r}A^{\frac{\delta_j}{\beta}}w_1\left|_{\left(\frac{\alpha_j+tm-\delta_j}{\beta_j},\frac{\delta_j}{\beta_j}\right)}^{---};j=1,...,r\right.\right]\right\}$$

$$\left\{H_{k-r,0}^{0,k-r}\left[\prod_{j=r+1}^{k}A^{-\frac{\delta_j}{\beta}}w_1\left|_{---}^{\left(1-\frac{\alpha_j+tm+\delta_j}{\beta_j},\frac{\delta_j}{\beta_j}\right)};j=r+1,...,k\right.\right]\right\} \quad (2.32)$$

For $\operatorname{Re}(\alpha_j + tm \pm (s-1)) > 0, s = v + 2r + 2, \delta_j > 0$.

For $\beta_j = 1 = \gamma_j; j = 1, ..., k$, the H-function reduces to the G-function.

Then the Hankel transform of $g(w_1)$ is given as:

$$H[w_1 J_v(pw_1)] = H\left[x_1^{\gamma_1} J_v(px_1^{\gamma_1})\right] ... H\left[x_r^{\gamma_r} J_v(px_r^{\gamma_r})\right]$$
$$H\left[x_{r+1}^{-\gamma_{r+1}} J_v\left(px_{r+1}^{-\gamma_{r+1}}\right)\right] ... H\left[x_k^{-\gamma_k} J_v\left(px_k^{-\gamma_k}\right)\right] \quad (2.33)$$

$$= J_v(p) \prod_{j=1}^{k} \frac{A^{\frac{\delta_j}{\beta_j}}}{\Gamma\left(\frac{\alpha_j + tm}{\beta_j}\right)} \left\{ \prod_{j=1}^{r} \frac{\Gamma\left(\frac{\alpha_j + tm + \delta_j(s-1)}{\beta_j}\right)}{A^{\frac{s\delta_j}{\beta_j}}} \right\} \left\{ \prod_{j=r+1}^{k} \frac{\Gamma\left(\frac{\alpha_j + tm - \delta_j(s-1)}{\beta_j}\right)}{A^{\frac{-s\delta_j}{\beta_j}}} \right\} \quad (2.34)$$

For $\operatorname{Re}(\alpha_j + tm \pm (s-1)) > 0, s = v + 2r + 2, \delta_j > 0$.

The unknown density $g(w_1)$ is obtained in terms of H-function by taking the inverse Hankel transform of (2.34). That is

$$g(w_1) = J_v(p) \prod_{j=1}^{k} \frac{1}{A^{\frac{-\delta_j}{\beta}} \Gamma\left(\frac{\alpha_j + tm}{\beta_j}\right)} \left\{ H_{0,r}^{r,0}\left[\prod_{j=1}^{r} A^{\frac{\delta_j}{\beta}} w_1 \middle|_{\left(\frac{\alpha_j + tm - \delta_j}{\beta_j}, \frac{\delta_j}{\beta_j}\right)}^{---}; j = 1, ..., r \right] \right\}$$

$$\left\{ H_{k-r,0}^{0,k-r}\left[\prod_{j=r+1}^{k} A^{-\frac{\delta_j}{\beta}} w_1 \middle|_{---}^{\left(1 - \frac{\alpha_j + tm + \delta_j}{\beta_j}, \frac{\delta_j}{\beta_j}\right)}; j = r+1, ..., k \right] \right\} \quad (2.35)$$

For $\operatorname{Re}(\alpha_j + tm \pm (s-1)) > 0, s = v + 2r + 2, \delta_j > 0$.

For $\beta_j = 1 = \gamma_j; j = 1, ..., k$, the H-function reduces to the G-function.

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