NANO AΨ-CLOSED SETS: A STUDY IN NANO TOPOLOGICAL SPACES

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ABSTRACT

In this article, we enclose the idea of nano alpha psi closed set in nano topological spaces and establish some of their properties. We also establish various forms of nano alpha psi locally closed, nano alpha psi locally closed star and nano alpha psi locally closed star star sets. Study some of related theorems.

Keywords: *Nαψ-closed set, NαψLC, NαψLC* and NαψLC*** .
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I. INTRODUCTION

O.Njastad [7] introduced the concept of α closed sets in topological spaces. M.K.R.S.Veerakumar[10] was introduced the notion of ψ closed sets. The notion αψ closed set in topological spaces are introduced by R.Devi et.al. [2]. Lellis Thivagar [3] was introduced a new concept of nano topology, it was defined interms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He formulated the notion of nano closed set, nano interior and nano closure and also he introduced the notion of nano semi closed and nano α closed sets. The concept of nano generalized locally closed sets in nano topology were introduced by K.Bhuvaneswari et.al. [1].

In aim of this paper we introduce the concept of nano αψ closed set and establish some of their properties. We also establish various forms of nano αψ Locally closed, NαψLC* and NαψLC** sets. Study some of related theorems.

II. PRELIMINARIES

We recall the following definitions, which are useful in the sequel.

Definition 2.1. [3] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X is a subset of U, then the lower approximation of X with respect to R is is denoted by $\underline{R} = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(X) denotes the equivalence class determined by $x \in U$.

Definition 2.2. [3] The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by $\overline{R} = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

Definition 2.3. [3] The boundary region of X with respect to R is the set of all objects, which can be possibly classified neither as X nor as not X with respect to R and its is denoted by $B_R = \overline{R} - \underline{R}$.

Definition 2.4. [3] If (U, R) is an approximation space and X, Y ⊆ U. Then

1. $\underline{R} \subseteq X \subseteq \overline{R}$
2. $\underline{R}(\phi) = \overline{R}(\phi) = \phi$ and $\underline{R}(U) = \overline{R}(U) = U$
3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
4. $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$
5. $\overline{R}(X \cup Y) \supseteq \overline{R}(X) \cup \overline{R}(Y)$

6. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$
7. $\overline{R}(X) \subseteq \overline{R}(Y)$ and $\underline{R}(X) \subseteq \underline{R}(Y)$ whenever $X \subseteq Y$
8. $\overline{R}(X^c) = (\underline{R})^c$ and $\underline{R}(X^c) = (\overline{R})^c$
9. $\underline{R}(\underline{R}(X)) = \overline{R}(\underline{R}(X)) = \underline{R}(X)$
10. $\overline{R}(\overline{R}(X)) = \underline{R}(\overline{R}(X)) = \overline{R}(X)$
11. R(X ∩Y) ⊆ R(X)∩R(Y)

Definition 2.5. [3] Let U be an universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, \overline{R}, \overline{R}, B_R\}$ where X ⊆ U. Then $\tau_R(X)$ satisfies the following axioms:
1. U and $\phi \in \tau_R(X)$.
2. The union of the element of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the element of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X. $(U,\tau_R(X))$ as the nano topological space. The element of $\tau_R(X)$ are called as nano open sets and complement of nano open sets is called nano closed.

Definition 2.6. [3] If $(U,\tau_R(X))$ is a nano topological spaces with respect to X where X ⊆ U and if A ⊆ U, then the nano interior of A is defined as the union of all nano open subsets contained in A and it is denoted by Nint(A). That is, Nint(A) is the largest nano open subsets contained in A and is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.

III. NANO αψ-CLOSED SETS

Definition 3.1. A subset H of $(U,\tau_R(X))$ is called nano αψ-closed set if ψcl(H) ⊆ V whenever H ⊆ V and V is nano α-open in $(U,\tau_R(X))$. The complement of nano αψ-closed set is nano αψ-open set.

Theorem 3.2. Each nano closed set is a nano αψ-closed set.

Proof. Let H be a nano α open set in $(U,\tau_R(X))$ and H ba a nano closed set is nano topological spaces $(U,\tau_R(X))$then A = Ncl(A). Every nano closed set is nano ψ closed set. Hence Nψcl(H) ⊆ Ncl(H) = H. Therefore Nψcl(H) ⊆ H. Hence H is Nαψ closed set.
The nano αψ-closed set is not nano closed set, this is proved by the following example.
Example 3.3. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology $\tau_R(X)$ = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o}, m,o},{l,m,n}, {m,n,o},{l,n,o}} and let A = {l}. Then A is not nano closed but it is nano αψ-closed set.

Theorem 3.4. Each nano closed set is a nano ψ-closed set.

Proof. Let H be a nano sg open set in $(U,\tau_R(X))$, and H ba a nano closed set is nano topological spaces $(U,\tau_R(X))$ then H = Ncl(H). Every nano closed set is nano semi closed set. Hence Nscl(H) ⊆ Ncl(H) = H. Therefore Nscl(H) ⊆ H. Hence H is Nψ closed set. The nano ψ-closed set is not nano closed set, this is proved by the following example.

Example 3.5. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology $\tau_R(X)$ = {U,φ,{l},{l,m,o},{m,o}}. Here NψCl = {U,φ,{l},{n},{l,n},{m,o},{m,n,o}} and let A = {m,n}. Then A is not nano closed but it is nano ψ-closed set.

Theorem 3.6. Each nano α closed set is a nano αψ-closed set.

Proof. Let H be a nano α open set in $(U,\tau_R(X))$, then H = Nαcl(H). Suppose H ⊆ V , V is Nα open set. Since H is Nα closed, Nψcl(H) ⊆ Nαcl(H) ⊆ V . Therefore H is Nαψ closed set. The nano αψ-closed set is not nano α closed set, this is proved by the following example.

Example 3.7. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology $\tau_R(X)$ = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o}, {l,m,n},{m,n,o},{l,n,o}} and let A = {n,o}. Then A is not nano α closed but it is nano αψ-closed set.

Theorem 3.8. Each nano semi closed set is a nano αψ-closed set.

Proof. Let H be a nano semi closed set in $(U,\tau_R(X))$, then H = Nscl(H). Suppose H ⊆ V , V is Nα open set. Since H is nano semi closed, Nψcl(H) ⊆ Nscl(H) ⊆ V . Therefore H is Nαψ closed set.
The nano αψ-closed set is not nano semi closed set, this is proved by the following example.
Example 3.9. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology $\tau_R(X)$ = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o},{l,m,n}, {m,n,o},{l,n,o}} and let A = {l,m,n}. Then A is not nano semi closed but it is nano αψ-closed set.

Theorem 3.10. Each nano sg closed set is a nano αψ-closed set.

Proof. Let H be a nano sg closed set in $(U,\tau_R(X))$. Suppose H ⊆ V , V is Nα open set. Every Nα open set is Nsemi open. Since H is nano semi closed, Nscl(H) ⊆ V then Nψcl(H) ⊆ Nscl(H) ⊆ V .Therefore H is Nαψ closed set. The nano αψ-closed set is not nano sg closed set, this is proved by the following example.

Example 3.11. Let U = {l,m,n,o} with U/R = {{l,m},{n},{o}} and X = {l,n}. Then the nano topology $\tau_R(X)$ = {U,φ,{n},{l,m,n},{l,m}}. Here NαψCl is power set and let A = {l,m,n}. Then A is not nano sg closed but it is nano αψ-closed set.

Theorem 3.12. Each nano gα closed set is a nano αψ-closed set.

Proof. Let H be a nano gα closed set in $(U,\tau_R(X))$. Suppose H ⊆ V , V is Nα open set. Since Nαcl(H) ⊆ V then Nψcl(H) ⊆ Nαcl(H) ⊆ V .Therefore H is Nαψ closed set. The nano αψ-closed set is not nano gα closed set, this is proved by the following example.

Example 3.13. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology $\tau_{R(}X)$ = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o},{l,m,n}, {m,n,o},{l,n,o}} and let A = {l,m,n}. Then A is not nano gα closed but it is nano αψ-closed set.
Theorem 3.14. Each nano ψ closed set is a nano αψ-closed set.

Proof. Let H be a nano ψ closed set in $(U,\tau_R(X))$. Suppose H ⊆ V , V is Nα open set. Every nano α open is nano semi open and every nano semi open set is nano sg open. Since Nscl(H) ⊆ V then Nψcl(H) ⊆ Nscl(H) ⊆ V .Therefore H is Nαψ closed set.
The nano αψ-closed set is not nano ψ closed set, this is proved by the following example.

Example 3.15. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology $\tau_R(X)$ = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o},{l,m,n}, {m,n,o},{l,n,o}} and let A = {m,n}. Then A is not nano ψ closed but it is nano αψ-closed set.

Theorem 3.16. Union of two nano αψ-closed sets are nano αψ-closed set.

Proof. Let F and G be two nano αψ-closed sets, Let E be any Nα open set in $(U,\tau_R(X))$, such that F ∪ G ⊆ E. Then F ⊆ E and G ⊆ E. Since F and G are nano αψ-closed set, Nψcl(F) ⊆ E and Nψcl(G) ⊆ E.Therefore Nψcl(F)∪Nψcl(G) = Nψcl(F ∪G) ⊆ E. Hence F ∪G is Nαψ-closed set.

Theorem 3.17. Let H be a subset and X is Nαψ-closed set in $(U,\tau_R(X))$then Nψcl(H)−H does not contain any empty Nα closed sets in $(U,\tau_R(X))$.

Proof. Let H is Nαψ closed sets and G be a non empty Nα closed subset of Nψcl(H)−H. Then G ⊆ Nψcl(H)∩(X −H). Since X −H is Nα open and H is Nαψ closed set. Nψcl(H) ⊆ X −H therefore G ⊆ X −Nψcl(H). Thus G ⊆ Nψcl(H)∩(X−Nψcl(H)) = φ. This implies that G = φ. Thus Nψcl(H)−H does not contain any non empty Nαψ closed.

Theorem 3.18. If F is Nαψ-closed set in U and F ⊆ G ⊆ Nαψcl(F) then G is also Nαψ-closed set in U. **Proof.** Suppose F is Nαψ-closed set in U. Let G ⊆ U such that V is Nα open set in U. Since F ⊆ G, we have F ⊆ V . Since F is Nαψ closed and Nψcl(G) ⊆ Nψcl(Nψcl(F)) = Nψcl(F) ⊆ V . Therefore Nψcl(G) ⊆ V . Hence G is Nαψ closed set in U.
The inverse of the above theorem need not true be true by the following example.

Example 3.19. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology τ_R(X) = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o},{l,m,n}, {m,n,o},{l,n,o}} and the set F = {l} and G = {l,n}. Such that F and G are Nαψ-closed sets but F ⊆ G but G is not a subset of Nαψcl(F).

Theorem 3.20. Let H is Nα-open and Nαψ-closed set then H is Nψ closed.

Proof. Since H ⊆ H and H is Nα open and Nαψ closed, we have Nψcl(H) ⊆ H. Thus Nψcl(H) = H. Hence H is Nψ closed set in U.

Theorem 3.21. A set H is Nαψ-open in (U,τ_R(X)) iff F ⊆ Nψint(H) whenever F is Nα closed in (U,τ_R(X)) and F ⊆ H.

Proof. Suppose F ⊆ Nψint(H) where F is Nα- closed and F ⊆ H. Let X − H ⊆ G where G is Nα open in (U,τ_R(X)). Then G ⊆ X − G and X − G ⊆ Nψint(H). Thus X − H is Nαψ-closed set in (U,τ_R(X)). Hence H is Nαψ-open in (U,τ_R(X)). Inversely, Suppose that H is Nαψ-open in (U,τ_R(X)). F ⊆ H and F is Nα closed in (U,τ_R(X)). Then X − F is Nα-open and X − H ⊆ X − F. Therefore Nψcl(X − H) ⊆ X − F. But Nψcl(X − H) = X − Nψint(H). Hence F ⊆ Nψint(H).

Theorem 3.22. A subset H is Nαψ-open in (U,τ_R(X)) iff G = X whenever G is Nα open and Nψint(H)∪(X −G) ⊆ G.

Proof. Let H be Nαψ-open. G be Nα-open and Nψint(H)∪(X−H) ⊆ G. This gives X −G ⊆ (X −ψint(H))∩(X −(X −H)) = X −Nψint(H)−(X −H) = Nψcl(X −H)−(X −H). Since X −H is Nαψ-closed and X −G is Nα-closed. Then by Theorem 3.21 it follows that X −G = φ. Therefore X = G. Inversely, Suppose F is Nα-closed and F ⊆ H. Then Nψint(H)∪(X −H) ⊆ (H)∪(X−F). It follows that ψint(H)∪(X−F) = X and hence F ⊆ Nψint(H). Therefore H is Nαψ-open in (U,τR(X)).

IV. NANO αψ-LOCALLY CLOSED SETS

We introduce the following definition.
Definition 4.1. Let H be a subset of nano topological spaces is called nano αψ locally closed sets (NαψLC) if H = K ∩R, where K is nano αψ open and R is nano αψ closed sets in nano topological spaces.

Definition 4.2. Let H be a subset of nano topological spaces is called nano αψ locally closed * sets (NαψLC*) if H = K∩R, where K is nano αψ open and R is nano closed sets in nano topological spaces.

Definition 4.3. Let H be a subset of nano topological spaces is called nano αψ locally closed sets (NαψLC**) if H = K∩R, where K is nano open and R is nanoαψ closed sets in nano topological spaces.

Example 4.4. Let U = {l,m,n,o} with U/R = {{l},{n},{m,o}} and X = {l,m}. Then the nano topology τ_R(X) = {U,φ,{l},{l,m,o},{m,o}}. Here NαψCl = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o},{l,m,n}, {m,n,o},{l,n,o}}.

Then NLC = {U,φ,{l},{n},{l,n},{m,o},{l,m,o},{m,n,o}} and 1. NαψLC is power set of U. 2.NαψLC* = {U,φ,{l},{m},{n},{o},{l,n},{l,m},{l,o},{m,o},{m,n,o},{l,m,o}}. 3.NαψLC** = {U,φ,{l},{m},{n},{o},{l,n},{m,n},{n,o},{m,o},{l,m,n}, {m,n,o},{l,n,o},{l,m,o}}

Remarks 4.5.
1. Let H be a subset of NT, is nano αψ locally closed iff $(U - H)^c$ is equal to union of all τR(x) and τ^c R(x).
2. Each nano αψ open subset of U is nano αψ locally closed sets and each nano αψ closed subset of U is nano αψ locally closed sets.
3. $[N\alpha\psi LC]^c$ is need not be a nano αψ locally closed sets.

Theorem 4.6. Each nano αψ closed set is nano αψ locally closed sets.

Proof. Obviously true by definition.

Theorem 4.7. A subset H of NT, 1. If H is nano locally closed, then H ∈ NαψLC (H ∈ NαψLC* and H ∈ NαψLC**) but inverse need not be true. 2. If H ∈ NαψLC* or H ∈ NαψLC**, then H is nano αψ locally closed.

Proof. This is proved by the following example.

Example 4.9.
1. Here all NLC set is NαψLC because it is power set of U, it is also NαψLC* and NαψLC**. It is proved obviously by Example [4.4].
2. This is also proved by Example [4.4] that is Each NαψLC* or NαψLC** is NαψLC set because it is power set of U.

Theorem 4.10. Let H be a subset of nano topology, then the following conditions are equivalent.
1. A subset H belongs to NαψLC*.
2. A subset H is equal to the intersection of both Nαψ open set K and nano closure of H.
3. NCL(H)−H is nano αψ closed.
4. Union of all H and U difference nano closure of H is nano αψ open.

Proof. (1)⇒(2) Suppose a subset H belongs to NαψLC* in nano topology. Then H = K∩R where K is Nαψ open and R is nano closed. Since H is a subset of K and H ⊆ Ncl(H), H ⊆ K ∩Ncl(H). Inversely, let H is a subset of R, Ncl(H) ⊆ R, we have H = K ∩R contains K ∩Ncl(H). Therefore K ∩Ncl(H) ⊆ H. Hence H is equal to the intersection of both Nαψ open set K and nano closure of H.

(2) ⇒ (1) Let K is Nαψ open and Ncl(H) is Nαψ closed K ∩Ncl(H) belongs to NαψLC*.
(2) ⇒ (3) Let a subset H is equal to the intersection of both Nαψ open set K and nano closure of H implies that $Ncl(H)-H = Ncl(H)\cap K^c$ which is Nαψ closed, since K^c is Nαψ closed.
(3) ⇒ (2) Suppose $K = [Ncl(H) - H]^c$, then by assumption K is Nαψ open in NT and H = K ∩Ncl(H).
(3) ⇒ (4) $H \cup (U - Ncl(H)) = H \cup (Ncl(H))^c = (Ncl(H)-H)^c$ and by (3) $(Ncl(H)-H)^c$ is Nαψ open and H ∪(U −Ncl(H)) is Nαψ open .
(4) ⇒ (3) Suppose $K = H \cup [Ncl(H)]^c$. Then Kc is Nαψ closed and $K^c = Ncl(H)-H$ and hence Ncl(H)−H is Nαψ closed.

Theorem 4.11. Let H be a subset of nano topology, then the following conditions are equivalent.
1. A subset H belongs to NαψLC set in nano topology.
2. Let H be a subset and which is equal to the intersection of both K for some Nαψ open set and nano αψ closure of H.
3. Nαψcl(H)−H is nano αψ closed.
4. Union of all H and $N\alpha\psi cl(H))^c$ is nano αψ open.
5. H is a subset of $N\alpha\psi int(H \cup (N\alpha\psi cl(H))^c)$.

Proof. (1) ⇒ (2) Let H belongs to NαψLC in nano topology. Then H = K∩R where K is Nαψ open and R is Nαψ closed. Since H is a subset of K and H ⊆ Nαψcl(H) implies H ⊆ K ∩Nαψcl(H). Inversely, let H is a subset of R, Nαψcl(H) ⊆ R and hence K∩Nαψcl(H) ⊆ H. Therefore K∩Nαψcl(H) ⊆ H. Hence H is equal to the intersection of both Nαψ open set K and nano closure of H.

(2) ⇒ (3) H = K ∩ Nαψcl(H) ⇒ Nαψcl(H) − H = Nαψcl(H) ∩ K^c which is Nαψclosed.
(3)⇒(4) H∩$(Nαψcl(H))^c$ = $(Nαψcl(H)−H)^c$ and by assumption $(Nαψcl(H)− H)^c$ is Nαψ open and it is H ∩$(Nαψcl(H))^c$.
(4) ⇒ (5) By assumption, H∪$(Nαψcl(H))^c$ = Nαψint(H∪(Nαψcl(H))c) and hence H ⊆ Nαψint(H ∩$(Nαψcl(H))^c$). (5) ⇒ (1) By assumption, H ⊆ Nαψcl(H), H = Nαψint(H ∪ $Nαψcl(H))^c$) ∩ Nαψcl(H) ∈ NαψLC in NT.

Theorem 4.12. A subset H of nano topology, then H belongs to NαψLC* iff H = K ∩ Nαψcl(H) for some nano open set K. Proof. Suppose H belongs to NαψLC** in nano topology. Then H = K∩R where K is nano open and R is Nαψ closed. Since, H ⊆ R, Nαψcl(H) ⊆ R. Now H = H∩Nαψcl(H) = K∩R∩Nαψcl(H) = K∩Nαψcl(H). Here the inverse part is true by definition.

Theorem 4.13. A subset H of nano topology. If H belongs to NαψLC** in nano topology, then Nαψcl(H)−H is Nαψ closed and H ∪(Nαψcl(H))c) is Nαψ open.

Proof. Suppose that H belongs to NαψLC** in nano topology. Then by Theorem [4.12], H = K∩Nαψcl(H) for some nano open K and Nαψcl(H)−H = Nαψcl(H)∩K^c is Nαψ closed. If R = Nαψcl(H)−H then R^c = H ∪$(Nαψcl(H))^c$ and R^c is Nαψ open and hence H ∪$(Nαψcl(H))^c$ is Nαψ open.

V. CONCLUSION

In this paper we have discussed about the new concept of nano closed set in nano topological spaces. We learn about the relationship between this set and already existing sets. Further we study the concept of nano αψ Locally closed, NαψLC* and NαψLC** sets and also derive some of their related properties.

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