

## Minimum Cross Entropy-Based Image Segmentation Using Innovative Optimization Techniques

Peter Daniel Smith

Assistant Professor, Department of Electronics and Communication Engineering, Cambridge Institute of Technology, Cambridge, UK

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### ABSTRACT

Image thresholding is an important technique for image processing and pattern recognition. Multilevel thresholding problem is often treated as a problem of optimization of an objective function. In this paper, minimum cross entropy (MCE) is introduced for multilevel thresholding which uses Improved Bacterial Foraging (IBF) algorithm for minimizing the MCE objective function. Some examples of test images are presented to compare the segmentation methods based on the IBF approach, with bacterial foraging (BF) algorithm, particle swarm optimization (PSO) algorithm and genetic algorithm (GA). From the viewpoint of visualization and image contrast, experimental results show that the thresholding method based on IBF method performs better than the BF, PSO and GA method. Moreover, the proposed method provides better accuracy, stability and computational efficiency.

*Keywords: Thresholding, Bacterial foraging algorithm, Optimization.*

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### I. INTRODUCTION

Segmentation in image processing finds immense application in various areas [1-3]. Thresholding is one of the most important techniques for performing image segmentation. It is typically simple and computationally efficient. However, the segmentation results depend heavily on the used image thresholding method. The image thresholding is widely used in infrared image segmentation, color image segmentation, halftone reproduction and mixed-type document analysis.

The main objective is to determine a threshold for bi-level thresholding or several thresholds for multilevel thresholding in image segmentation. Bi-level thresholding selects only one threshold which separates the pixels into two classes, while multilevel thresholding determine multiple thresholds which divide the pixels into several groups. There are many image thresholding studies in the literature over the years.

Kapur et al. [4] used the concept of the entropy of a histogram and developed a global thresholding method separating the histogram of gray level probabilities into two distributions of the image. Yin [5] proposed a property based method which first developed a peak finding method based on symmetry; then, the duality property was used to identify the valleys of the histogram. Perhaps, the most important and widely accepted concept is on the characteristics of thresholding techniques. Sahoo et al. [6] presented a comprehensive survey of a variety of thresholding techniques and Abutaleb [7] classified them into parametric and non-parametric approaches.

Parametric approaches assume each grouping having the probability density function of a Gaussian distribution and find an estimate of the parameters of such distribution which will best fit the given histogram data [8]. Unfortunately, when the desired number of classes is much lower than the number of peaks in the original histogram, the computation time to find the solutions of threshold values often becomes expensive. In contrast, non-parametric approaches find the threshold level according to some discriminating a criterion such as between-class variance Otsu [9] and entropy Kapur et al. [4] which both separates the gray level regions of an image in an optimum manner. As the result, the non-parametric approaches are proven to be more computationally efficient and similar to apply.

Despite the fact that the problem of thresholding has been quite extensively studied many years, the automatic determination of an optimum threshold value continues to be of great challenge. One method to find the thresholds is an exhaustive search, which means calculating the objective function for every possible placement of the thresholds. The problem with this approach is that when the image is segmented into more than two classes, the time needed to find the optimal thresholds increases dramatically with the number of gray levels and the number of classes. One way to overcome this limitation is to perform an iterative algorithm.

Lim and Lee [10] proposed a method which first smoothes the grey-level histogram by the Gaussian convolution. He detected the multiple thresholds by computing the first and second derivatives of the smoothed histogram. Then again, the computation time grows exponentially with the number of thresholds due to its exhaustive searching strategy, which would limit the multilevel thresholding applications.

Another alternative to fast multilevel thresholding uses a new class of algorithms, called intelligent algorithms. Intelligent algorithms play an important role in computer science, artificial intelligence, operational research, and other related fields [11].

In recent years, several heuristic optimization techniques such as Ant Colony Optimization (ACO), Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) were introduced into the field of image segmentation because of their fast computing ability. Several techniques using genetic algorithms (GAs) have been proposed to solve the multilevel thresholding problem [12], [13].

Though GA-based approaches perform well for complex optimization problems, recent research has identified certain deficiencies [14], particularly for problems in which variables are highly correlated. In such cases, the GA crossover and mutation operators do not generate individuals with better fitness of offspring as the chromosomes in the population pool have some structure towards the end of the search.

Besides GA, particle swarm optimization (PSO) is another latest evolutionary optimization technique which is used for multilevel thresholding [15] [16]. In Zahara et al., [16], the PSO is used in conjunction with the simple method for the Gaussian curve fitting and for the Otsu's function optimization.

In this paper, a novel approach, Improved Bacterial Foraging (IBF) algorithm is proposed to solve the multilevel thresholding problem in image segmentation. In order to verify its feasibility, the proposed method is tested on ten standard test images and the results are compared with BF, PSO and GA methods to demonstrate its performance. The results indicate the applicability of the proposed method to the multilevel thresholding problem.

The rest of the paper is organized as follows. The multilevel thresholding problem formulation is described in section II. In section III, the proposed IBF algorithm is explained. Section IV presents the simulation results and comparison with those of other methods. Finally, in section V, conclusions are drawn based on the results found from the simulation analyses in section IV.

## II. PROBLEM FORMULATION OF MINIMUM CROSS ENTROPY THRESHOLDING

The cross entropy was proposed by Kullback [17]. Let  $P = \{p_1, p_2, p_3 \dots p_N\}$  and  $Q = \{q_1, q_2, q_3 \dots q_N\}$  be the two probability distributions on the same set. The cross entropy  $P$  and  $Q$  is information theoretic distance between the two distributions and it is defined by

$$D(p, q) = \sum_{i=1}^N p_i \ln \frac{p_i}{q_i}$$

The minimum cross entropy thresholding algorithm selects the thresholds by minimizing the cross entropy between the original image and its thresholded version.

Let there be  $L$  gray levels in a given image and these gray levels are in the range  $\{0, 1, 2, \dots, (L-1)\}$ ,  $I$  be the original image and  $h(i) = 0, 1, 2 \dots L$  be the corresponding histogram. Then the resulting image, denoted by  $I_t$  using  $t$  as the thresholded value that is constructed by

$$I_t(x, y) = \begin{cases} \mu(I, t), & I(x, y) < t \\ \mu(t, L+1), & I(x, y) \geq t \end{cases}$$

where

$$\mu(a, b) = \frac{\sum_{i=a}^{b-1} ih(i)}{\sum_{i=a}^{b-1} h(i)}$$

The cross entropy for bi-level thresholding is then calculated by

$$\text{Min } D(t) = D_0 + D_1$$

where,

$$D_0 = - \sum_{i=0}^{t-1} i h(i) \log \left( \frac{\sum_{i=0}^{t-1} i h(i)}{\sum_{i=0}^{t-1} h(i)} \right)$$

$$D_1 = - \sum_{i=t}^L i h(i) \log \left( \frac{\sum_{i=t}^L i h(i)}{\sum_{i=t}^L h(i)} \right)$$

This MCE thresholding method has also been extended to multilevel thresholding and can be described as follows: The optimal multilevel thresholding problem can be configured as a  $m$ -dimensional optimization problem, for determination of  $m$  optimal thresholds for a given image  $[t_1, t_2 \dots t_m]$ , where the aim is to minimize the objective function:

$$\text{Min } D(t_0 + t_1 + t_2 \dots + t_m) = D_0 + D_1 + D_2 \dots + D_m \tag{1}$$

where,

$$D_0 = - \sum_{i=0}^{t_1-1} i h(i) \log \left( \frac{\sum_{i=0}^{t_1-1} i h(i)}{\sum_{i=0}^{t_1-1} h(i)} \right)$$

$$D_1 = - \sum_{i=t_1}^{t_2-1} i h(i) \log \left( \frac{\sum_{i=t_1}^{t_2-1} i h(i)}{\sum_{i=t_1}^{t_2-1} h(i)} \right)$$

$$D_2 = - \sum_{i=t_2}^{t_3-1} i h(i) \log \left( \frac{\sum_{i=t_2}^{t_3-1} i h(i)}{\sum_{i=t_2}^{t_3-1} h(i)} \right).$$

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and

$$D_m = - \sum_{i=t_m}^L i h(i) \log \left( \frac{\sum_{i=t_m}^L i h(i)}{\sum_{i=t_m}^L h(i)} \right)$$

The minimum cross entropy thresholding method is very efficient in bi-level thresholding cases. However, its computational time becomes aggravated in the case of multilevel thresholding. To make the multilevel MCE thresholding method more practical in image segmentation, this paper proposes MCE threshold selection based on IBF algorithm. The aim of this proposed method is to minimize the MCE thresholding objective function using equation (1).

### III. PROPOSED IBF ALGORITHM

In this paper, optimization using improved bacterial foraging algorithm is proposed to find the optimal thresholds in multilevel thresholding problem. The BF algorithm proposed by Passino [19] is modified to expedite the convergence. The modifications are discussed below. The minimum value of objective function for each bacterium, in the chemotactic steps in any generation, is saved before sorting is done for reproduction. Instead of taking the average of all the chemotactic cost functions for deciding the healthiest bacteria, the minimum as proposed above is adapted, i.e. the bacterium having the minimum cost function is retained for the next generation. For swarming, the distances of all the bacteria in a new chemotactic stage is evaluated from the global optimum bacterium till that point and not the distances of all the bacteria from rest others as suggested in ref. [19]. The improved bacterial foraging algorithm is discussed in Figure 1.

**BEGIN**

Initialize the parameter,  $C(i)$ ,  $i = 1, 2, \dots, S$ . Also initialize all the counter values to zero.

**REPEAT**

**For**  $l = 1$  to  $N_{ed}$

**For**  $K = 1$  to  $N_{re}$

**For**  $j = 1$  to  $N_c$

**For**  $i = 1$  to  $S$

Compute  $J(i, j, k, l)$

Then let  $J(i, j, k, l) = J(i, j, k, l) + J_{cc}^i(\theta^i(j+1, k, l), P(j, k, l))$

$J_{last} = J(i, j, k, l)$

Tumble: Generate a random vector  $\Delta(i) \in \mathfrak{R}^P$

Move:  $\theta^i(j+1, k, l) = \theta^i(j, k, l) + C(i) \frac{\Delta(i)}{\sqrt{N}}$

Compute  $J(i, j+1, k, l)$  Then let  $J(i, j+1, k, l) = J(i, j+1, k, l) + J_{cc}^i(\theta^i(j+1, k, l), P(j+1, k, l))$

$m = 0$

#### IV. COMPUTATIONAL RESULTS AND ANALYSIS

The proposed minimum cross entropy (MCE) thresholding based improved bacterial foraging (IBF) algorithm is implemented in MATLAB under a personal computer with 3GHz CPU, and 2 GB RAM in windows XP system. In the implementation of IBF algorithm, it needs many parameters that are predefined. Table I shows the parameters of IBF algorithm. Ten test images named lena, pepper, baboon, hunter, map, cameraman, living room, house, airplane and butterfly are used for conducting the experiments. These original test images are shown in Fig. 2 along with their histograms.

**Table I**  
**parameters used for ibf method**

Parameter	Value
Number of bacterium ( $s$ )	20
Number of chemotatic steps ( $N_c$ )	10
Swimming length ( $N_s$ )	10
Number of reproduction steps ( $N_{re}$ )	4
Number of elimination of dispersal events ( $N_{ed}$ )	2
Depth of attractant ( $d_{attract}$ )	0.1
Width of attract ( $\omega_{attract}$ )	0.2
Height of repellent ( $h_{repellent}$ )	0.1
Width of repellent ( $\omega_{repellent}$ )	10

Probability of elimination and dispersal ( $P_{ed}$ )	0.02
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To test the effectiveness of the proposed algorithm, it is compared with BF, PSO and GA methods. Table II summarizes the objective values obtained by all the four algorithms. Since the considered objective function is a minimization problem, the minimum value indicates the better performance of the algorithm. From table II, it is observed that the IBF algorithm provides better performance than the other three methods. Furthermore, the objective function decreases exponentially with the number of required thresholds.



Fig. 2 Test Images



Fig. 3 Thresholded images obtained by MCE based IBF method

**Table II**  
the objective values and standard deviation values obtained by mce based four multilevel thresholding methods

Test Images	Objective values $\times (10^9)$				Standard Deviation			
	IBF	BF	PSO	GA	IBF	BF	PSO	GA
Lena	10.7132	10.7132	10.7132	10.7132	0.0000	0.0000	0.0000	0.0000
	7.9371	7.9376	8.1331	8.4895	1.9145e+007	2.4828e+007	1.4028e+008	2.5106e+008
	6.4848	6.4852	6.7606	7.1689	2.4571e+007	2.9046e+007	1.5260e+008	2.8948e+008
	5.4003	5.4113	5.8280	6.2674	3.0014e+007	3.1495e+007	2.0828e+008	4.7978e+008
Pepper	2.8420	2.8420	2.8420	2.8420	0.0000	0.0000	0.0000	0.0000
	2.05011	2.0504	2.0518	2.1164	1.3300e+006	1.4300e+006	1.4146e+007	1.0806e+007
	1.6114	1.6265	1.7093	1.8113	6.5218e+006	6.6532e+006	2.8183e+007	4.3821e+007
	1.3610	1.3693	1.4226	1.5008	8.5111e+006	8.6984e+006	3.2069e+007	1.0156e+008

**Table III**  
optimal threshold values obtained by mce based algorithms

Test Images	No. of thresholds	Optimal threshold values			
		IBF	BF	PSO	GA
LENA	2	110,165	110,165	110,165	110,165
	3	105,140,189	94,138,178	87,134,174	94,138,178
	4	80,117,161,195	76,116,152,187	64,113,145,181	67,107,145,183
	5	75,114,135,160,191	63,107,136,162,197	65,93,135,168,195	61,103,124,169,194
PEPPER	2	110,174	110,174	110,174	110,174
	3	95,140,184	95,144,186	92,144,184	87,131,179

4	71,116,162,197	77,116,161,191	74,115,151,189	75,101,148,196
5	68,104,136,169,197	60,108,147,175,197	63,96,131,169,202	60,93,129,168,206

In order to find the stability of all the algorithms, the standard deviations are also calculated for all the algorithms and are summarized in Table II. From the results, the standard deviation value of IBF algorithm is lesser than the BF, PSO and GA which illustrates the better stability of the proposed IBF algorithm. Table III summarizes the number of thresholds and the corresponding optimal threshold values obtained by all the four algorithms. Fig. 3 is the segmented images by the proposed MCE based IBF algorithm for various threshold levels. It is clearly seen from the figure that the quality of the segmented image is better when the number of thresholds  $m = 5$  is chosen for all the images. The performance matrices for checking the effectiveness of the method are chosen as the computation time so as to get an idea of complexity and peak to signal noise ratio (PSNR) measure which is used to determine the quality of the thresholded images. Table IV furnishes the number of thresholds, PSNR measure and the corresponding CPU time taken by all the algorithms.

The PSNR value obtained by the proposed IBF method is higher than the BF, PSO and GA methods. Furthermore, as the number of thresholds increase, the PSNR value is raised. It is also perceived from table that the CPU time taken by the proposed method is shortest, which shows the better convergence speed of the algorithm.

**Table IV**  
*the psnr measure and the corresponding cpu time*

Test Images	No. of thresholds	PSNR (db)				CPU time (Seconds)			
		IBF	BF	PSO	GA	IBF	BF	PSO	GA
LENA	2	15.2352	15.2352	15.2352	15.2352	8.1800	8.1875	9.5781	10.2031
	3	17.5480	17.5483	17.4893	17.3556	8.3384	8.3438	9.9219	10.8281
	4	19.4012	19.3910	19.0003	18.6737	9.4511	9.4688	10.3438	11.1406
	5	21.5168	21.5078	21.1539	20.5928	10.2618	10.2500	10.9401	11.8871
PEPPER	2	14.5835	14.5835	14.5835	14.5835	7.7581	7.7581	8.7969	9.4375
	3	16.2679	16.2467	16.1968	16.1593	8.4213	8.4375	9.6094	10.0000
	4	18.6321	18.5793	18.4858	18.1006	9.7554	9.7581	10.4375	10.9435
	5	20.5280	20.5222	20.0197	19.7713	10.7192	10.7085	11.0625	12.0469

## V. CONCLUSION

Although the minimum cross entropy thresholding method is very efficient in bi-level thresholding cases, its computational time becomes aggravated in the case of multilevel thresholding. To make the multilevel MCE thresholding method more practical in image segmentation, this paper proposes MCE thresholding selection based on IBF algorithm. For testing the effectiveness of the algorithm, it considers ten standard test images. From the experimental results, it is observed that the proposed MCE thresholding based IBF algorithm (i) is more efficient to search the near optimal solutions compared to the BF, PSO and GA methods, (ii) the quality of the segmented images by the proposed method is much superior than the other algorithms, and (iii) the proposed algorithm provides better stability. Furthermore, experiments on a variety of images show that the new algorithm efficiently segments the image in computationally less time.

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