# NON-LINEAR BEHAVIOUR OF MASONRY WALLS: FE, DE & FE/DE MODELS

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**Abstract**

Non-linear behaviour of masonry panels is a topic of great interest in civil engineering and architecture fields. Several numerical approaches may be found in literature.

Here, three different models are presented and compared to investigate non-linear behaviour of in-plane loaded masonry walls: Discrete Element (DE) model, combined Finite-Discrete Element (FE/DE) model, Finite Element model based on a total rotating strain smeared crack approach (FE-TRSCM). Hence, analysis of masonry is carried out at different scales to compare reliability and application field of the models.

DE and FE/DE models adopt a micro-modelling strategy based on discrete cracks, blocks modelled as independent bodies and mortar joints as elastoplastic Mohr-Coulomb interfaces. These approaches already turned out to be in good agreement for in plane non-linear analysis. Here, FE/DE model adopts hypothesis of infinitely resistant and deformable blocks, with cracks occurring only along mortar joints. Deformability is assumed in the triangular FE domain discretization and embedded crack elements may be activated whether tensile or shear strength is reached.

FE-TRSCM follows a macro-modelling approach based on smeared crack theory, often adopted for concrete. Masonry is modelled as a homogeneous material, with a yield criterion based on fracture energy accounting for masonry softening response in compression and tension.

Three approaches are compared and calibrated by reproducing experimental tests on masonry panels in compression and under an increasing shear action. The parametric analyses show the capacity and limit of local micro-models or continuous diffused model to represent masonry behaviour.

**Keywords**
masonry; discrete micro- model; continuous macro- model; total rotating strain crack model; pushover analysis.

## 1. Introduction

Masonry is a composite structural material made of natural or artificial resisting elements connected by dry or mortar joints. Masonry has been adopted for load-bearing purposes since the beginning of human stable civilization. Due to the large amount of masonry historical buildings that may be found everywhere, especially in southern Europe, together with the frequent seismic events in that area, the assessment of the behaviour of masonry and the development of numerical models for masonry elements or buildings is an active field of research in civil engineering.

Among different numerical strategies that are frequently adopted for studying masonry and that may be found in literature, this work focuses on discrete element (DE) models, combined finite-discrete element (FE/DE) models and finite element (FE) models based on a total rotating strain crack approach (FE-TRSCM). The first two models may be considered micro or local models, while the third may be considered macro or diffused model. A comparative analysis between models allows to perform a multi-scale analysis of masonry behaviour, this latter is the most important task in masonry study. Masonry is a particular composite in which two scales coexist: block micro-scale, panel macro-scale.

DE and FE/DE models are based on several similar hypotheses that characterise the wide field of discrete modelling. In particular, these models are characterised by the definition of the elements and the contacts between the elements themselves. Furthermore, following the original definition of the method (Cundall, 1971; Cundall and Hart, 1985, 1992), a discrete approach considers specimens composed by independent elements, that may undergo large displacements; then, if the contacts between the elements vary during the numerical tests, the contact topology can be updated. Due to the material heterogeneity, typical of masonry, discrete approaches may be successfully adopted for studying this material and several models based on the discrete method, originally conceived for rock mechanics, may be found in literature, together with several case studies (Lemos, 2007).

The DE model adopted here is mainly suitable for studying historical masonry, since it considers blocks as rigid bodies and dry or mortar joints as elastoplastic interfaces. It is based on the original contribution by Cecchi and Sab (2004), based on small displacements hypothesis. This contribution has been recently extended to the field of material non-linearity accounting for a Mohr-Coulomb yield criterion for restraining interface actions and considering an elasto-perfectly-plastic interface behaviour (Baraldi and Cecchi, 2016).

The FE/DE model adopted here was originally introduced in soil mechanics problems (Munjiza et al., 1995; Munjiza, 2004); however, it has been successfully extended to the field of masonry structures by authors (Reccia et al., 2012) and by other research groups (Smoljanović et al., 2013, 2015). Furthermore, the DE and FE/DE models adopted here have been already calibrated for modelling masonry panels with dry joints in-plane loaded (Baraldi et al., 2015, 2017), showing a good agreement between the models.

Then, focusing on FE models for masonry, many different models have been proposed by researchers up to current days and two main approaches may be considered: micro and macro modelling (Lourenço at al., 1995). The first approach is a standard heterogeneous approach, with joints and bricks modelled as different materials by specific FEs, and joints often modelled by interface elements (Page, 1978; Lourenço and Rots, 1997). In the second approach, a homogeneous material equivalent to the heterogeneous one is considered and modelled by a unique type of FE. For masonry material and structures, the existing macro-models can be distinguished between those specifically developed for masonry with respect to those originally introduced for concrete and then extended to masonry.

In this field of models, homogenization approaches may be adopted by identifying a periodic cell typical of the masonry material and by adopting local linear and non-linear mechanical parameters (De Buhan and de Felice, 1997; Luciano and Sacco, 1997; Milani 2011; Bertolesi et al., 2016), also by performing multi-scale analyses (Addessi and Sacco, 2012). In particular, multi-scale approaches are focused on Cauchy or micropolar/non-local continuum models (Stefanou et al., 2008; Salerno and de Felice, 2009; De Bellis and Addessi, 2011; Bacigalupo and Gambarotta, 2011; Baraldi and Cecchi, 2014). Another possibility in the field of continuous modelling is an approach based on the evaluation of the overall properties of a masonry assemblage by fitting experimental tests. Orthotropic and also isotropic damage models are often adopted for this purpose (da Porto et al., 2010). Focusing on damage models, discrete or smeared crack approaches may be adopted by extending existing models originally conceived for concrete. The discrete crack approach considers the cracks as zero thickness interfaces elements, and the crack opening results as mesh discontinuity. Modelling the cracks by interface elements has few limitations into continuous models, like the necessity to induce cracking to follow a default path along the edges of the FE model and to redefine continuously the mesh during the crack opening and evolution. The smeared crack approach is an energy-based method in which cracks do not depend on the geometry, hence FE mesh redefining is not required. The smeared crack models have been firstly used for concrete numerical modelling simulations (Rashid, 1968). A smeared crack approach can be developed by multi-directional or total strain approaches. Multi-directional approach is based on the strain decomposition into an elastic and inelastic part and cracks may assume different orientations (de Borst and Nauta, 1985; Rots and Blaauwendraad, 1989). Total strain approach is based on the stress-strain constitutive relationship and it is fixed or rotating, depending if the crack orientation remains constant (fixed) or continuously rotates with the principal strains axes (rotating).

The continuous total strain crack model (TSCM) is a proper tool for modelling strengthened masonry structures (Ghiassi et al., 2013; Gattulli et al., 2014; Wang et al., 2017) and homogenous isotropic materials as rammed earth masonry (Micolli et al., 2015; Haach et al., 2014). Moreover, it has also been adopted for modelling unreinforced masonry panels under in-plane loads (da Porto et al., 2010; De Carvalho et al., 2017).

This work is dedicated to the assessment of the non-linear behaviour of masonry panels with regular texture subject to vertical and horizontal in-plane actions. The aim of this work is to critically evaluate the similarities and the differences between the three modelling choices introduced previously, focusing firstly on the input parameters and then on the output results. Attention is initially paid to the calibration between the numerical models for correctly simulating both the linear and the non-linear behaviour of masonry. Starting from the calibration already done by the authors for DE and FE/DE models with reference to the case of masonry panels with dry joints, this calibration is extended to the case of mortar joints. Then, particular attention is devoted to the elastic and inelastic parameters necessary to define an appropriate homogeneous material based on the continuous total rotating strain crack approach (FE-TRSCM). For this purpose, the experimental campaign by Raijmakers and Vermeltfoort (1992) on almost square-shaped masonry panels is taken as reference, together with the work of Lourenço and Rots (1997), that studied the same panels by means of FE models with interface elements.

Differently from the previous comparisons between DE and FE/DE models, which were focused on linear elastic behaviour (Baraldi et al., 2013) and on non-linear behaviour in case of dry joints (Baraldi et al., 2015; Baraldi et al., 2017), in this case masonry specimens with mortar joints are considered, hence non-zero tensile strength and cohesion are taken into account for mortar. Moreover, in FE/DE, bricks are modelled as elastic deformable bodies, while in the previous applications they were modelled as rigid blocks. It is worth noting that the experimental campaign simulated numerically is characterised by different levels of vertical compression, that may cause compressive failure of bricks and mortar joints. These types of failure are not modelled by the DE and FE/DE models and this aspect may represent a limitation for the correct assessment of masonry behaviour compared with the results of experimental tests.

The paper is organised as follows: the adopted models are briefly introduced, focusing on their differences and similarities; then, the case study considered is described and the results of the numerical tests performed with the adopted models are showed and deeply discussed, accounting also for experimental and other numerical results. Finally, several considerations are done by highlighting the main advantages and disadvantages of each model adopted and by evaluating possible further developments.

## 2 Basic assumptions

In this work, three different numerical models are adopted, compared and critically evaluated. DE and FE/DE models are refined models able to take into account the actual texture of the masonry specimens and they adopt the mechanical parameters of blocks and mortar, whereas the FE-TRSCM approach requires less mechanical parameters, to be determined by means of homogenization procedures on the basis of experimental results and/or detailed models.

The principal topic is how a macro-diffused-model as FE-TRSCM may take into account local micro mechanical characteristic of a composite as the masonry material?

### *2.1 DE micro-model*

As stated in the introduction, the DE model adopted here is an improvement of the original contribution by Cecchi and Sab (2004), that proposed a simple and effective model based on rigid blocks connected by elastic interfaces in a running bond pattern. This micro-model allowed to perform dynamic analyses by adopting a molecular dynamics solution method, that is typically used by other discrete approaches. Such model has been improved by Baraldi and Cecchi (2016, 2017a) both for the in- and out-of-plane loading case, by adding an elastoplastic behaviour at interface level and by restraining interface actions with a Mohr-Coulomb yield criterion. More recent developments regard the extension of model applicability to the case of random or quasi-periodic masonry (Baraldi and Cecchi, 2017b).

The main advantage of this model is represented by the small number of degrees of freedom involved in the numerical analyses of masonry panels, given that only the translations of block centres and the block rotations with respect to their centres are considered as the unknowns of the problem (Fig. 1a). The elastic deformability of the model is governed by the mortar elastic modulus $E_m$ and Poisson's ratio $\nu_m$, that allow to define the normal, shear and rotational stiffness of the interfaces as follows:

$$K_n = E_m \cdot A_i / e_i, \quad K_s = G_m \cdot A_i / e_i, \quad K_r = E_m \cdot I_i / e_i, \tag{1a-c}$$

where $A_i$, $I_i$ and $e_i$ are, respectively, interface area, inertia and thickness.

A local coordinate system at interface level (Fig. 2a) is assumed. Hence interface actions, that are represented by normal and shear forces $F_\perp$ and $F_\parallel$, together with an interface bending moment $M$ for the in-plane case, are defined as the resultants of interface normal and shear stresses, $\sigma_\perp$ and

$\sigma_{||}$, and depend on relative displacements (normal and tangential translations $d_{\perp}$, $d_{||}$ and rotation $d_{\theta}$) between adjacent blocks:

$$F_{\perp} = K_{\perp} \cdot d_{\perp}, \quad F_{||} = K_{||} \cdot d_{||}, \quad M = K_{\theta} \cdot d_{\theta} \, . \tag{2a-c}$$

The non-linear behaviour of the model is governed by the Mohr-Coulomb yield criterion, written in terms of stress resultants, but still characterised by a cohesion $c$ and a friction angle $\varphi$, together with the interface tensile strength $f_t$:

$$F_{\perp} \le F_{\perp,u} = f_t \cdot A_i, \quad | F_{||} | \le F_{||,u} = f_t \cdot A_i - F_{\perp} \tan\varphi, \quad | M | \le M_u = (F_{\perp,u} - F_{\perp}) \cdot l_{c,i} \, , \tag{3a-c}$$

where $l_{c,i}$ represents the characteristic length of the interface, that is equal to one half of interface length.

Compressive failure is not taken into account, together with block deformability and block failure. An elastoplastic behaviour for mortar joints is assumed: when interface actions reach the corresponding elastic limit, governed by tensile strength and Mohr-Coulomb criterion, the corresponding stiffness are set equal to zero.

Due to the assumption of small displacements, leading to a fixed contact topology of the elements, this model does not exactly follow the typical hypotheses of discrete approaches, but it allows to adopt a static solution method with the determination and update of the stiffness matrix of the system, in order to quickly perform pushover analysis of masonry walls and façades. However, the model does not allow to study entire buildings or large building portions, due to the huge number of degrees of freedom that may be involved in the numerical tests. This formulation, furthermore, will allow to update the model accounting also for large displacements.

*2.2 FE/DE micro-model*

The combination of FE models with DE models had already been introduced in the initial improvements of the original discrete models, in order to describe the deformability of the elements by means of simple FE discretizations (Cundall and Hart, 1985, 1992). The FE/DE model adopted here, as stated in the introduction, is based on the original code developed by Munjiza and co-workers (Munjiza et al., 1995). Numerical tests have been performed by means of the open source computer code Y2D/Y-GUI (Mahabadi et al., 2010) for generating input files and Y-Geo (Mahabadi et al., 2012) for running numerical analyses. Masonry specimens are discretised with a mesh of triangular FEs in plane stress. In order to obtain a model closer to the DE one, the mesh discretization should be characterised by forces and/or displacements applied at nodes located along the centroidal axis of the blocks (Fig. 1b). Block elastic deformability is then governed by its elastic modulus and Poisson's ratio, whereas the non-linear behaviour is obtained by means of zero thickness crack elements that are embedded between all the triangular FEs. Crack elements can represent actual mortar joints of the masonry specimen, but they can also represent inner block subdivisions, allowing cracks to develop both into the blocks and between adjacent blocks. This aspect represents an advantage of the FE/DE model with respect to the simpler DE model, but it requires a larger computational effort with respect to the DE model, due to the larger number of degrees of freedom represented by nodal translations of each triangular element. It is worth noting that the FE/DE model follows the typical hypotheses of discrete models, allowing to perform analyses with large displacements and to update the contact topology of each element, without requiring the determination of the stiffness matrix of the entire specimen considered. In fact, with this model, dynamic analysis can be effectively performed, with the possibility to evaluate load multipliers and collapse mechanisms, whereas pushover analyses can be performed only by assessing each load step separately.

Interfaces are generally characterised by a tensile strength criterion ($\sigma_{\perp} \leq f_t$) and Mohr-Coulomb yield criterion for restraining shear stresses ($|\sigma_{||}| \leq c - \sigma_{\perp} \cdot \tan\varphi$). Such criterions are assumed equal to those adopted in the DE model, whereas the inelastic behaviour is further detailed with mode I and mode II fracture energies for tensile and shear failure, respectively:

$$G_I = (l_i \cdot \pi \cdot f_t^2) / E_m, \quad G_{II} = (l_i \cdot \pi \cdot c) / E_m, \qquad (4a,b)$$

where $l_i$ is interface length.

However, in the following numerical tests, blocks are assumed to be infinitely resistant to tensile and shear forces, then damage can occur only along the crack elements between adjacent blocks, in order to obtain a behaviour close to that of the simpler DE model previously described.

### *2.3 FE macro-model based on total rotated strain smeared crack approach, FE-TRSCM*

As stated in the introduction, the FE-TRSCM is a macro-modelling approach originally introduced for studying concrete structures. In this model, masonry is assumed as a continuous and its local mechanical characteristic are considered diffused. However, this model has been frequently adopted for masonry elements, building portions and entire buildings, in particular in the case of strengthening interventions, allowing to model several aspects such as masonry irregularity and tensile fragility, together with inner or outer reinforcements. For this reason, this model may be less effective in modelling unreinforced masonry, since it does not take into account the anisotropic effects due to the mortar joints.

In particular, the fixed and rotating crack approaches are suitable for modelling material damage assuming the cracks occur into one principal direction. However, damage in brittle materials, such as concrete and masonry subject to in-plane tensile failure, is often characterised by parallel cracks aligned along a main direction. Then, in this contribution, the simplification of rotating approach would be appropriate.

Assuming masonry modelled as an isotropic equivalent continuum, described by its elastic modulus $E$ and its Poisson's ratio ν, a damaged plasticity model is adopted for representing its non-linear behaviour, with reference to a global coordinate system $y_1, y_2$ (Fig. 2b). This model is based on the incremental plasticity theory and on the concept of isotropic damage elasticity to describe the irreversible damage that occurs during the fracturing process. Incremental plasticity relates plastic stress increments $\{\Delta\sigma_{11}\ \Delta\sigma_{22}\ \Delta\sigma_{12}\}^T$ with plastic strain increments $\{\Delta\varepsilon_{11}\ \Delta\varepsilon_{22}\ \Delta\gamma_{12}\}^T$ by means of a constitutive matrix that adopts a softening post-crack parameter $\mu$ and a shear retention factor $\beta$:

$$\begin{Bmatrix}\Delta\sigma_{11}\\ \Delta\sigma_{22}\\ \Delta\sigma_{12}\end{Bmatrix}=\begin{bmatrix}\mu E/(1-\nu^2\mu) & \nu\mu E/(1-\nu^2\mu) & 0\\ & E/(1-\nu^2\mu) & 0\\ sym & & \beta E/[2(1+\nu)]\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_{11}\\ \Delta\varepsilon_{22}\\ \Delta\gamma_{12}\end{Bmatrix}. \qquad (5)$$

Different failure mechanisms take place: crushing in compression and cracking in tension. An exponential softening law in tension is considered, whereas a parabolic law in compression is adopted. Both constitutive laws are characterised by yield stresses ($f_t$ in tension and $f_c$ in compression), and by the corresponding fracture energy ($G_t$, $G_c$), that need to be considered with the corresponding dimensionless values with respect to finite element size $h$:

$$G_t = G_I / h, \quad G_c = G_C / h, \qquad (6)$$

where $G_C$ is obtained by integrating the ellipsoid law for cap model in compression adopted by Lourenço and Rots (1997).

In the following, models and analyses are performed using the multipurpose FE code DIANA (TNO DIANA), by defining a regular mesh of quadrilateral elements in plane stress state (Fig. 1c). The parameters of the constitutive models are calibrated on the basis of the experimental results.

## 3 Numerical experimentation

*3.1 Specimen characteristics*

The experimental tests over almost-square-shaped masonry panels performed by Raijmakers and Vermeltfoort (1992) are taken as reference ones for the numerical experimentation of this work. Furthermore, the numerical simulation of the cited tests performed by Lourenço and Rots (1997) is taken as reference for several mechanical parameters to be adopted. The masonry panels are composed by blocks having the following dimensions: width $b$ = 0.204 m, height $a$ = 0.05 m, thickness $s$ = 0.098 m. Mortar joint thickness is $e$ = 0.01 m, both for head (vertical) and bed (horizontal) joints. Blocks are arranged in a running bond pattern, with four and one half blocks in horizontal direction, and 18 courses in vertical direction, leading to specimen overall dimensions $L$= 0.99 m and $H$ = 1.00 m (Fig. 3). Panels are fixed along their bases by means of a first steel beam, whereas they are loaded by a second steel beam along their upper edge. This beam neglects vertical relative translations of the upper edge of each panel and it is also used for loading each panel with three different levels of compression: $p$ = 0.30 – 1.21 – 2.12 MPa. Furthermore, an increasing horizontal displacement at the upper edge is applied, in order to activate the shear resistance of each panel, strictly depending on the compressive level. It is worth noting that, following Lourenço and Rots (1997), several mechanical parameters regarding mortar non-linear behaviour turn out to depend on the compressive level. Fig. 4 shows the crack patterns obtained at the end of the experimental tests, with cracks occurring both along mortar joints and into the blocks.

*3.2 Numerical model parameters*

Focusing on geometric parameters, the DE model requires only the coordinates of block centres together with block and interface dimensions, in order to define the stiffness and compatibility matrices of the assemblage (Baraldi and Cecchi, 2016); then, the specimen considered is characterised by 240 degrees of freedom. The FE/DE model adopts a mesh refinement characterised by 16 triangular FEs in plane stress for each block, leading to an overall number of 1152 FEs for the panel and 6192 degrees of freedom. It is worth noting that block dimensions, in this case, accounts also for mortar joint thickness, given that the FE/DE model adopts zero-thickness interfaces. The FE-TRSCM adopts a mesh of 1600 quadrilateral eight-noded elements in plane stress, with 13122 degrees of freedom, not strictly dependent on block dimensions and arrangements.

The mechanical parameters adopted by the three numerical models are resumed in Tab. 1, by distinguishing the linear elastic parameters with respect to the non-linear ones. In particular, the DE model requires the definition of mortar elastic modulus and Poisson’s ratio, that are determined starting from the interface stiffness values adopted by Lourenço and Rots (1997). In this case, it is worth noting that the elastic deformability of the model accounts for mortar deformability only, without averaging its elastic modulus with that of blocks. The non-linear parameters for the DE model are assumed equal to the Mohr-Coulomb parameters of mortar joints. Similarly, the FE/DE model adopts the same Mohr-Coulomb parameters, together with the fracture energy values for fracture modes I and II, as suggested by Lourenço and Rots (1997). In this contribution, the elastic deformability is taken into account into the FE/DE model by considering a homogenised elastic modulus and a Poisson's ratio $E$, $\nu$ for masonry material. The FE-TRSCM adopts the same elastic parameters of the FE/DE model, in order to have the same elastic deformability. Similarly, the tensile behaviour is governed by the same tensile strength $f_t$ adopted by DE and FE/DE models, together with the tensile – or type I – fracture energy $G_t$. For the compressive behaviour, the compressive strength adopted for the cap model by Lourenço and Rots (1997), whereas the compressive fracture energy is evaluated by assuming a stress-strain behaviour in compression described by an ellipsoid curve.

*3.3 Numerical test results*

An incremental analysis for each masonry panel is performed by reproducing the experimental tests. It is worth noting that the DE model and the FE-TRSCM are able to perform an actual pushover analysis with displacement control, whereas the FE/DE model is able to perform independent analyses for each load or displacement increment.

Starting with the first specimen, compressed by $p$ = 0.30 MPa, Fig.5a shows shear force-horizontal displacement curves obtained with the three different numerical models, together with experimental testing results. All the models are able to follow the initial elastic branch of the experimental curve and to reach the order of magnitude of the maximum shear force reached by the panel, with slightly different softening branches. In particular, the curve obtained with the FE/DE model is characterised by several sudden load variations for small displacement increments also in the elastic branch, that is typical of actual discrete approaches, whereas the curve obtained with the DE model is characterised by this aspect after the elastic branch, due to the local failure of mortar interfaces. Fig. 6 shows the collapse mechanisms obtained with the three models. Both mechanisms obtained with DE and FE/DE models are characterised by tensile failure of almost all horizontal mortar joints close to the upper and lower steel beam, together with a tensile and shear failure of inner mortar joints, generating a diagonal crack that follows the texture of the specimen. The mechanism obtained with the FE model is characterised by a diagonal band of cracks. Panel behaviour is showed in Fig. 7 by the crack and damage patterns obtained with the tree models. Comparing experimental (Fig. 4a) and numerical results, DE and FE/DE models turn out to correctly simulate the tensile failure of horizontal joints close to the steel beams, whereas the diagonal cracks do not follow the position and direction obtained experimentally. The FE-TRSCM is able to simulate the development of a diagonal crack that does not follow actual masonry pattern and, with this load level, horizontal tensile cracks close to the steel beams are not obtained.

Considering then the second specimen, compressed by $p$ = 1.21 MPa, Fig. 5b shows shear force-horizontal displacement curves obtained with the three different numerical models together with experimental testing results. Similarly to the previous case, all models are in good agreement with experimental results, with DE and FE/DE model results still characterised by sudden load variations with small increments of displacements. The DE model is able to reach larger displacements with respect to the other models, even if the maximum shear force is obtained with a large displacement value with respect to experimental results and other numerical models. Fig. 8 shows the collapse mechanisms obtained with the tree models. Similarly to the previous case, both mechanisms obtained with DE and FE/DE models are characterised by tensile failure of almost all horizontal mortar joints close to the upper and lower steel beam, whereas in this case the tensile and shear failure of inner mortar joints generates different diagonal cracks. In DE model two main diagonal cracks in the upper and lower portions of the panel may be found, whereas in the FE/DE model, a unique crack following the diagonal of the panel is obtained. The mechanism obtained with the FE model is characterised by a diagonal band of cracks and by the compressive failure of the materials at the upper-right and lower-left corners. As previously done, in Fig. 9, the crack and damage patterns are given with reference to the different approaches adopted. Comparing results with respect to experimental test results (Fig. 4b), in this case DE and FE/DE models turn out to correctly simulate the diagonal cracking behaviour, but they are unable to model the compressive failure of mortar joints at corners, together with cracking of several bricks at corners and along the diagonal of the panel. For this reason, in this case, the damage pattern showed by the FE-TRSCM is closer to the experimental results.

Finally, Fig. 5b shows shear force-horizontal displacement curves obtained with the three different numerical models together with experimental testing results, for the third specimen, compressed by $p$ = 2.12 MPa. In this case, due to the high compressive level that causes compressive collapse of mortar joints and bricks, the FE-TRSCM turns out to be very close to experimental results, both for the maximum shear force and for the softening branch of the load-displacement curve. DE and FE/DE models slightly overestimate the maximum shear load and do not follow the softening behaviour of the specimen, due to the hypothesis of infinitely resistant blocks and to the absence of a compressive strength for mortar interfaces. Collapse mechanisms (Fig. 10) and crack patterns (Fig. 11) are given also in this case for the three models. DE and FE/DE model results are similar to those obtained with the previous cases, with horizontal cracks close to the steel beams and more diffuse diagonal cracks into the panel. Results obtained with the FE-TRSCM are characterised by a bigger diagonal crack with respect to the previous cases, together with the compressive failure of more elements close to the upper-right and lower-left corner, due to the higher compression applied to the specimen.

## Conclusions

In this work, an experimental campaign on masonry panels, undergoing shear and compressive loading, has been modelled by adopting three different numerical approaches.

A DE micro-model with rigid blocks and elastoplastic interfaces, together with a combined FE/DE micro-model with elastic blocks, and interfaces (subject to tensile and shear failure and contact detection algorithm), have been compared to a continuous macro-model represented by a FE based on a total rotating strain crack approach (TRSCM).

This work answers the question of how a macro-diffused-model as FE-TRSCM may take into account local micro mechanical characteristic of a composite of masonry.

The numerical tests have shown that both discrete micro (DE and FE/DE) and continuous macro (FE-TRSCM) models represent effective tools for modelling masonry structures and, in particular, to investigate the non-linear behaviour of masonry walls in-plane loading conditions. The discrete (DE and FE/DE) models are able to take into account the real local texture of masonry walls, to predict with accuracy the real crack pattern and the potential mechanisms of collapse. The main objective of local micro models is to calibrate the mechanical parameters for the FE diffused continuous model. The principal limitation of local micro models is the computational charges for large-scale models.

In general, continuous FE models find application on the modelling of large-scale masonry structures. Focusing on the smeared crack approach, the continuous total rotating strain crack model (TRSCM) represents a simplified model able to predict the principal crack directions, the main deformation mechanisms and the collapse loads of masonry panels subject to combined shear and compressive actions. A limitation of this model is represented by the isotropic behaviour of the homogeneous material, that will be overcome in future developments of this work, by assuming an orthotropic material behaviour based on the actual masonry texture. Another possible development of this work may regard the application of these numerical approaches to study the response of strengthened masonry panels; best effectiveness is expected from the FE-TRSCM approach while existing DE or FE/DE models needs to be deeply modified to take into account reinforcements.

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## References

Addessi, D., Sacco, E., A multi-scale enriched model for the analysis of masonry panels, *Int. J. Solids Struct.*, vol. 49, no. 6, pp. 865-880, 2012.

Bacigalupo, A., Gambarotta, L., Non-Local Computational Homogenization of Periodic Masonry, *Int. J. Multiscale Comp. Eng.*, vol. **9**, no. 5, pp. 565-578.

Baraldi, D., Cecchi, A., Discrete Element Model for in-Plane Loaded Viscoelastic Masonry, *Int. J. Multiscale Comp. Eng.*, vol. **12**, no. 2, pp. 155-175, 2014.

Baraldi, D., Cecchi, A., Discrete approaches for the nonlinear analysis of in plane loaded masonry walls: Molecular dynamic and static algorithm solutions, *Eur. J. Mech. A/Solids*, vol. **57**, pp. 165-177, 2016.

Baraldi, D., Cecchi, A., A full 3D rigid block model for the collapse behaviour of masonry walls, *Eur. J. Mech. A/Solids*, vol. 64, pp. 11-28, 2017.

Baraldi, D., Cecchi, A., Discrete model for the collapse behavior of unreinforced random masonry walls, *Key Eng. Mat.*, vol. **747**, pp. 3-10, 2017.

Baraldi, D., Reccia, E., Cazzani, A., Cecchi, A., Comparative analysis of numerical discrete and finite element models: the case of in-plane loaded periodic brickwork, *Compos Mech. Comp. Appl. Int. J.*, vol. **4**, pp. 319-344, 2013.

Baraldi, D., Reccia, E., Cecchi, A., DEM & FEM/DEM models for laterally loaded masonry walls, *COMPDYN 2015 - 5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, pp. 2144-2157, 2015.

Baraldi, D., Reccia, E., Cecchi, A., In plane loaded masonry walls: DEM and FEM/DEM models. A critical review, *Meccanica*, in press, pp. 1-16, 2017. DOI: 10.1007/s11012-017-0704-3

Bertolesi, E., Milani, G., Lourenço, P.B., Implementation and validation of a total displacement non-linear homogenization approach for in-plane loaded masonry, *Comput. Struct.*, vol. **176**, pp. 13-33, 2016.

Cecchi, A., Sab, K., A comparison between a 3D discrete model and two homogenized plate models for periodic elastic brickwork, *Int. J. Solids Struct.*, vol. **41**, pp. 2259-2276, 2004.

Cundall, P.A., A computer model for simulating progressive large scale movements in blocky rock systems, *Symposium of International Society of Rock Mechanics*, Nancy, France, 1971.

Cundall, P.A., Hart, R.D., Development of generalized 2-d and 3-d distinct element programsfor modeling jointed rock. ITASCA Consulting Group, final report, 1985.

Cundall, P.A., Hart, R.D., Numerical modelling of discontinua, *Eng. Comput.*, vol. **9**, no. 2, pp. 101-113, 1992.

da Porto, F., Guidi, G., Garbin, E., Modena, C., In-Plane Behavior of Clay Masonry Walls: ExperimentalTesting and Finite-Element Modeling, *J. Struct. Eng.*, vol. **136**, no. 11, pp. 1379-1392, 2010.

De Bellis, M.L., Addessi, D., A Cosserat Based Multi-Scale Model for Masonry Structures, *Int. J. Multiscale Comp. Eng.*, vol. **9**, no. 5, pp. 543-563, 2011.

de Borst, R., Nauta, P., Non-orthogonal cracks in a smeared finite element model, *Eng. Comput.*, vol. **2**, pp. 35-46, 1985.

De Buhan, P., De Felice, G., A homogenization approach to the ultimate strength of brick masonry, *J. Mech. Phys. Solids.*, vol. 45, no. 7, pp. 1085-1104, 1997.

De Carvalho, C., Cecchi, A., Meroi, E., Oliveira, D.V., Experimental and Numerical Investigations on the Behaviour of Masonry Walls Reinforced with an Innovative Sisal FRCM System, *Key Eng. Mat.*, vol. 747, pp. 190-195, 2017.

Gattuli, V., Lampis, G., Marcari, G., Paolone, A., Simulations of FRP reinforcement in masonry panels and application to a historic façade, *Eng. Struct.*, vol. **75**, pp. 604–618, 2014.

Ghiassi, B., Oliveira, D.V., Lourenço, P.B., Marcari, G., Numerical study of the role of mortar joints in the bond behavior of FRP-strengthened masonry, *Compos. Part B Eng.*, vol. **46**, pp. 21-30, 2013.

Haach, V.G., Vasconcelos, G., Lourenço, P.B., Study of the behaviour of reinforced masonry wallets subjected to diagonal compression through numerical modelling, *9th International Masonry Conference*, Guimarães, 2014.

Lemos, J.V., Discrete Element Modeling of Masonry Structures, *Int. J. Archit. Herit.*, vol. **1**, pp. 190-213, 2007.

Lourenço, P.B., Rots, J.G., Blaauwendraad, J., Two approaches for the analysis of masonry structures: micro and macro-modeling, *Heron.*, vol. **40**, no. 4, 1995.

Lourenço, P.B., Rots, J.G., Multisurface interface model for analysis of masonry structures, *J. Eng. Mech.*, vol. **123**, no. 7, pp. 660-668, 1997.

Luciano, R., Sacco, E., Homogenization technique and damage model for old masonry material, *Int. J. Solids Struct.*, vol. **34**, no. 24, pp. 3191-3208, 1997.

Mahabadi, O.K., Grasselli, G., Munjiza, A., Y-GUI: A graphical user interface and preprocessor for the combined finite-discrete element code, Y2D, incorporating material inhomogeneity, *Comput. Geosci.*, vol. **36**, pp. 241-252, 2010.

Mahabadi, O.K., Lisjak, A., Munjiza, A., Grasselli, G., Y-Geo: a new combined finite discrete element numerical code for geomechanical applications, *Geomech.*, vol. **12**, pp. 676–688, 2012.

Miccoli, L., Oliveira, D. V., Silva, R.A., Müller, U., Schueremans, L., Static behaviour of rammed earth: Experimental testing and finite element modelling, *Mat. Struct.*, vol. **48**, no. 10, pp. 3443-3456, 2015.

Milani, G., Simple homogenization model for the non-linear analysis of in-plane loaded masonry walls, *Comput. Struct.*, vol. **89**, pp. 1586-1601, 2011.

Munjiza, A., Owen, D.R.J., Bicanic, N., A combined finite-discrete element method in transient dynamics of fracturing solids, *Eng. Comput.*, vol. **12**, no. 2, pp. 145-174, 1995.

Munjiza, A., *The finite/discrete element method*, Chicester: John Wiley and Sons, 2004.

Page, A.W., Finite Element Model for Masonry, *J. Struct. Div.*, vol. **104**, pp. 1267-1285, 1978.

Raijmakers T.M., Vermeltfoort, A.T., Deformation controlled meso shear tests on masonry piers. Rep. B-92-1156, TNO-BOUW/TU Eindhoven, Build. and Constr. Res., Eindhoven, Netherlands, 1992.

Rashid, Y.R., Analysis of prestressed concrete pressure vessels, *Nucl. Eng. Des.*, vol. **7**, no. 4, 334-355, 1986.

Reccia, E., Cazzani, A., Cecchi, A., FEM-DEM Modeling for Out-of-plane Loaded Masonry Panels: A Limit Analysis Approach, *Open Civ. Eng. J.*, vol. **6**, no.1, pp. 231-238, 2012.

Rots, J.G., Blaauwendraad, J., Crack models for concrete: Discrete or smeared, fixed, multi-directional or rotating, *Heron*, vol. **34**, no. 1, pp. 1-59, 1989.

Salerno, G., De Felice, G., Continuum modeling of periodic brickwork. *Int. J. Solids Struct.*, vol. **46**, no. 5, pp. 1251-1267, 2009.

Smoljanović, H., Zivaljic, N., Nikolic, Z., A combined finite-discrete element analysis of dry stone masonry structures, *Eng. Struct.*, vol. **52**, pp. 89-100, 2013.

Smoljanović, H., Nikolić, Z., Živaljić, N., A combined finite-discrete numerical model for analysis of masonry structures, *Eng. Fract. Mech.*, vol. **136**, pp. 1-14, 2015.

Stefanou, I., Sulem, J., Vardoulakis, I., Three-dimensional Cosserat homogenization of masonry structures: elasticity, *Acta Geotech.*, vol. **3**, pp. 71-83, 2008.

TNO DIANA, DIANA. DIsplacement method ANAlyser, release 9.4, User's Manual.

Wang, X., Ghiassi, B., Oliveira, D.V., Lam, C.C., Modelling the nonlinear behaviour of masonry walls strengthened with textile reinforced mortars, *Eng. Struct.*, vol. **134**, pp. 11-24, 2017.

## Figure captions

Fig. 1. Numerical models adopted for studying in-plane masonry behaviour: (a) DE, (b) FE/DE and (c) FE models.

Fig. 2. Local coordinate system at interface level for DE and FE/DE models (a); global coordinate system at specimen level for FE-TRSCM (b).

Fig. 3. Masonry specimen considered for the numerical tests.

Fig. 4. Crack patterns obtained at the end of the experimental tests.

Fig. 5. Load-displacement curves for the panel subject to different compressive levels: (a)$p$ = 0.30 MPa, (b) p = 1.21 MPa, (c) p = 2.12 MPa.

Fig. 6. Failure mechanisms for masonry panels subject to $p$ = 0.30 MPa and modelled with (a) DE model, (b) FE/DE model, (c) FE-TRSCM.

Fig. 7. Crack and damage patterns at maximum shear force for masonry panels subject to $p$ = 0.30 MPa and modelled with (a) DE model, (b) FE/DE model, (c) FE-TRSCM.

Fig. 8. Failure mechanisms for masonry panels subject to $p$ = 1.21 MPa and modelled with (a) DE model, (b) FE/DE model, (c) FE-TRSCM.

Fig. 9. Crack and damage patterns at maximum shear force for masonry panels subject to $p$ = 1.21 MPa and modelled with (a) DE model, (b) FE/DE model, (c) FE-TRSCM.

Fig. 10. Failure mechanisms for masonry panels subject to $p$ = 2.21 MPa and modelled with (a) DE model, (b) FE/DE model, (c) FE-TRSCM.

Fig. 11. Crack and damage patterns at maximum shear force for masonry panels subject to $p$ = 2.12 MPa and modelled with (a) DE model, (b) FE/DE model, (c) FE-TRSCM.

## Table captions

Tab. 1. Mechanical parameters adopted by the numerical models.